INTERACTIONS BETWEEN ECONOMY AND CLIMATE:

A FRAMEWORK FOR POLICY DESIGN UNDER UNCERTAINTY

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I. INTRODUCTION

It is now widely recognized that economic activities affect the world's climate, and that there is at least some possibility that these effects may be sufficiently substantial to be a cause of serious concern. Many mechanisms have been suggested as carriers of the impact of economic activities on the climate. Prominent among these are the emission of carbon dioxide (CO_2) as a result of the combustion of fossil fuels, the emission of waste heat, particularly from power generation, the contamination of the atmosphere with aerosols and particulates, and changes in the ecological balance as a result of agricultural activities.[1]

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The generation of CO_2 and of aerosols and particulates is thought to change the reflective and absorbtive characteristics of the atmosphere for electromagnetic radiation, thus possibly changing the rates at which the earth heats and cools. The emission of waste heat may, like intensive agricultural activities, change the local climate, and if this is done on a sufficiently large scale in geographic terms there is a possibility of such effects becoming global.

Although there is general agreement that all of these effects *may* provide mechanisms whereby economic activities affect the climate, there is unfortunately substantial uncertainty about the magnitudes and characteristics of these effects. To be precise, there is uncertainty on several counts.

1. The *time scale* on which climate might respond to economic activity is not clear. There is general agreement that none of the effects concerned is likely to be immediate, but beyond this there is little common ground. Some commentators see a chance of significant responses in this century, whereas others see them at least half a century distant.
2. There is also a wide range of opinions about the possible magnitudes and directions of economic impacts on the climate. It is not clear whether we should expect increases or decreases in global mean temperatures, variance of temperatures, precipitation, etc.
3. Finally, there is little information about the effects of possible climate changes on human activity in general, and on economic activity in particular. It is not clear whether any particular change is likely to be harmful or beneficial to agricultural activities, or to enforce major and costly changes in patterns of settlement or energy use.

It might be helpful to illustrate briefly the types of uncertainty involved. Clearly one of the most important mechanisms for interactions between economy and climate is the emission of CO_2 into the atmosphere, with its possible effects on the transmission and absorption of electromagnetic radiation. The main source of such emission is the combustion of fossil fuels: CO_2 is released into the atmosphere by combustion and removed from it by photosynthesis and by solution into the oceans. In order to make a forecast as to the effects of CO_2 release, it is clearly necessary to predict:

1. The use of fossil fuels
2. The rate of removal of CO_2 from the atmosphere
3. The effect of changes in CO_2 concentration on climate
4. The economic effects of climate changes

A little reflection reveals that every one of these tasks is clearly very difficult. The first—the prediction of fossil fuel consumption—is perhaps the simplest, yet most energy economists would attach little confidence to any prediction of energy consumption more than 10 or 15 years hence. One has to predict gross national product (GNP), the relationship between GNP and energy consumption, and the shares of different fuel types in supplying this consumption. Most existing long-run predictions of energy consumption are based essentially on an extrapolation of existing energy–GNP relationships and so of course lead to the prediction that as the currently less-developed countries industrialize, energy consumption, and hence fossil fuel consumption and so the release of CO_2, will rise greatly on a worldwide basis. However, recent studies of energy consumption in the Third World[2] have suggested that official statistics for many poor countries greatly understate the true levels of energy consumption because of the extensive reliance on energy sources such as firewood, dung, and animal power, none of which is recorded in official energy statistics. This omission will of course bias *upward* the apparent relationship between energy consumption per head and GNP per head because of the underreporting of energy consumption at low income levels and will also bias *downward* estimates of current emission of CO_2 into the atmosphere. Such climatic effects as are observed will thus be attributed to less CO_2 than is actually released, overestimating the impact of CO_2. It thus emerges that a simple shortcoming in official statistics—a shortcoming of a kind that is very common—may give a misleading appearance to the problem.

I have so far discussed only the first of the four links in the chain of reasoning needed to predict the effect of CO_2 release on economic activities via its effect on the climate. It is clear that even this, almost surely the simplest link, involves great uncertainties. This is quite sufficient for the present purpose, which is merely to emphasize that our knowledge of the structure of the problem with which we are concerned is extremely partial and imperfect. It is therefore of great importance that any decision making in the field of economy–climate interactions should be carried out in the light of an explicit recognition of the immense uncertainties involved.

The remaining sections of this paper attempt to raise some of the issues involved in giving due recognition to these uncertainties. I present two very simplified models of stochastic economy–climate interaction and derive optimal decision rules for both of them. Neither can claim to be more than extremely partial; nevertheless they appear to demonstrate that the relevant issues can be formalized quite tractably.

Both models are dynamic and have as a central feature the fact that the *timing* of any response of climate to economic activity is very uncertain but that this timing is at least partially controllable via the level of economic activity. Thus we do not know when a given sustained rate of release of CO_2 into the atmosphere will lead to major changes, but we do know that, at least in expected value terms, lower rates of fossil fuel burns will delay the changes.

The first of the two models, discussed in Section II, is very simple and hence admits a complete analytical solution. The second, given in Section III, has analytical solutions only for very special cases. The model of Section II is based on the idea that the atmosphere has a limited capacity to absorb emissions such as CO_2 and particulates before undergoing a major change. However, this capacity is unknown; we only discover it when we reach the limit and notice the changes. The only information we have a priori is a probability distribution over possible values of this absorbtive capacity. The model then addresses the question: given this probability distribution, how should we control our emission-producing activities?[3]

The second model, discussed in Section III, is considerably more complex. It is supposed that the climatic environment may be in one of two states, either *favorable* or *unfavorable* to economic activity. It is initially in the favorable state, and it may move stochastically to the unfavorable state with probabilities that depend on the history of economic activities persued to date. Given that the dependence of these probabilities on economic activities is a known function, the question naturally arises: how should the economic activities be controlled over time?

In both cases, then, there is an uncertain dependence of irreversible climatic changes on cumulative economic activities. The uncertainty is described by a probability distribution on a stochastic process, and the issue is how the relevant economic activities should be controlled over time.

Before beginning the formal analysis, it is perhaps appropriate to comment on the role and value of the highly simplified models that are about to be discussed. Clearly they tackle the issues involved only at a very high level of abstraction and simplification. I think it can be argued that this is an essential characteristic of a first step in this area: one has to pick out the basic issues, see how to formalize them, and determine whether they are tractable. The issue, in this case, is not so much whether the model is too simple as whether the simplications are the right ones. If they are, then the models analyzed below can be used as "building blocks" in the construction and analysis of more complete and integrated pictures of the overall problem. They can also be used to highlight the key relationships and parameters, so that we know where it is most important to improve our knowledge.

II. UNCERTAIN ABSORBTIVE CAPACITY

It is supposed that the atmosphere has a total capacity A to absorb emissions. As far as the decision makers are concerned, A is an unknown random variable with a marginal density function f(A):

$$f(A) \geqslant 0, \qquad \int_0^\infty f(A)\, dA = 1.$$

Here A is not, of course, truly a stochastic variable: in principle its value can be known, so the uncertainty facing decision makers stems from a lack of information rather than from the random nature of the environment. Once the total absorbtive capacity A has been utilized, then there is a major irreversible and harmful change in the climate—a true catastrophe. The analysis does not require a precise specification of this change, so that the reader may be left to exercise a vivid imagination on this issue.

It is supposed that the rate of emission at time t is E_t, a nonnegative real number, and that there are benefits of value $V(E_t)$ associated with a rate of emission of E_t. The value of the catastrophe when it occurs, measured in the same units as benefits, is C. The overall decision-making problem thus has as an objective:

$$\text{maximize expectation} \left\{ \int_0^T V(E_t) e^{-\delta t}\, dt + C e^{-\delta T} \right\}, \tag{1}$$

where $\delta \geqslant 0$ is a discount rate to be applied to future benefits; T is the date at which the total absorbtive capacity A is exhausted and at which the catastrophe occurs, and the expectation is over all possible values of T; $V(E_t)$ is of course a continuous real-valued increasing function and will usually be assumed to be strictly concave. On occasions it will however be of interest to investigate the impact of certain threshold effects which can be represented by a nonconcavity in V(E).

The structure of the problem can now be set out more formally. The choice variable is E, the emission rate, as a function of time, E_t. Any particular emission policy, say $\hat{E}_t$, taken in conjunction with the density function f(A) describing possible values of A, induces a probability distribution over possible values of T, the date at which absorbtive capacity is exhausted and the climatic catastrophe occurs. This distribution is given by

$$\text{proby}(T = T') = f\left(\int_0^{T'} \hat{E}_t\, dt \right). \tag{2}$$

For convenience we let

$$Z_t = \int_0^t E_\tau \, d\tau, \qquad \frac{dz}{dt} = \dot{Z}_t = E_t, \tag{3}$$

so that (2) becomes

$$\text{proby}(T = T') = f(Z_t).$$

The problem (1) can now be restated as

$$\max \int_0^\infty f(Z_t) \left\{ \int_0^T V(E_t)e^{-\delta t} + Ce^{-\delta T} \right\} dZ_t \tag{4}$$

subject to $\dot{Z}_t = E_t$ [i.e., Eq. (3)].

The maximization of (4) subject to (3) is now a well-defined dynamic optimization problem,[4] which may be solved as follows. Rewrite the maximand as

$$\int_0^\infty f(Z_t) \frac{dz}{dt} \left\{ \int_0^T V(E_t)e^{-\delta t} + Ce^{-\delta T} \right\} dt$$

$$= \int_0^\infty f(Z_t)E_t \left\{ \int_0^T V(E_t)e^{-\delta t} + Ce^{-\delta T} \right\} dt.$$

We now simplify the problem by choosing the origin of the payoff function so that

$$C = 0.$$

This is clearly an acceptable normalization. Defining

$$\int_T^\infty f(Z_t)E_t \, dt \equiv \int_{Z_T}^\infty f(Z_t) \, dZ = F(Z_t)$$

and integrating by parts, the maximand becomes

$$\int_0^\infty V(E_t)e^{-\delta t} \left\{ \int_T^\infty f(Z_t)E_t \, dt \right\} dt$$

$$= \int_0^\infty V(E_t) \, F(Z_t)e^{-\delta t} \, dt. \tag{5}$$

The relevant optimization problem is now that of maximizing (5) subject to the differential Eq. (3). This may readily be solved by, for example, Pontryagin's maximum principle. Letting p_t be the shadow price or co-state variable associated with the constraint, the Hamiltonian is

$$H = V(E) \, F(Z)e^{-\delta t} + pEe^{-\delta t}$$

and the conditions necessary for a solution are, assuming an interior solution,

$$V'(E) \, F(Z) = -p, \tag{6}$$

where $V'(E) = dV/dE$ and

$$\dot{p} - \delta p = -V(E) \, F'(Z). \tag{7}$$

Notice from (6) that as V(E) is an increasing function, $p \leq 0$. Intuitively the state variable Z is a "bad" rather than a "good" variable and so has a negative shadow price.

$$F'(Z_t) = \frac{d}{dZ_t} \int_{Z_T}^\infty f(Z_t) \, dZ = -f(Z_t).$$

Differentiation of (6) leads to

$$\frac{V''E}{V'} \frac{\dot{E}}{E} - \frac{fE}{F} = \frac{\dot{p}}{p},$$

which in combination with (7) gives

$$\frac{V''E}{V'} \frac{\dot{E}}{E} - \frac{fE}{F} = \delta + \frac{V}{V'} \frac{F'}{F}. \tag{8}$$

Defining

$$y(E) = \frac{V''E}{V'}$$

and

$$q(Z) = \frac{f(Z)}{F(Z)}$$

so that q(Z) is the conditional probability of the change occurring at cumulative emission level Z, given that it has not occurred previously, (8) may be simplified to

$$\frac{\dot{E}}{E} = \frac{\delta}{y(E)} + \frac{q(Z)}{y(E)} \left\{ E - \frac{V(E)}{V'(E)} \right\}.$$

This, in turn, can conveniently be rewritten in final form as

$$\frac{\dot{E}}{E} = \frac{\delta}{y(E)} + \frac{q(Z)}{y(E)} . \frac{E.}{V'} \left\{ V'(E) - \frac{V(E)}{E} \right\}. \tag{9}$$

This is a differential equation governing the optimal rate of change of emissions over time. For any strictly concave function, we have

$$y(E) < 0$$

and

$$V'(E) < \frac{V(E)}{E},$$

as for any such function the marginal payoff $V'(E)$ is strictly less than the average $V(E)/E$. Hence for a strictly concave function the first term in the expression for $\dot{E}/E$ is negative and the second positive. This second term tends to zero as $q(Z)$, the conditional probability of the catastrophe, tend to zero.

It is clear from (9) that $E = 0$ if and only if

$$E\left\{\frac{\delta}{y} - \frac{qV}{yV'} + \frac{q}{y}E\right\} = 0.$$

Thus $E = 0$ is a locally stable stationary solution, with a second stationary solution given by

$$E = \frac{V}{V'} - \frac{\delta}{q}.$$

As V, V', and q are not in general constants, $E = 0$ is in general the only stationary solution.[5] Hence a solution to the overall optimization problem will consist of picking a time path for E which converges to zero according to (9). It is clear from this that the rate of decline of E is greater, the greater is the discount rate δ applied to future benefits and the nearer is the payoff function to linear.

A ready intuitive interpretation can be provided for these results. The atmosphere has a limited (though uncertain) absorptive capacity, and an increase in the discount rate represents a change in preferences toward using up that absorptive capacity now rather than later: it hence tends to produce an emission path which starts higher but falls faster than would be the case with a lower discount rate. It is worth noting that, in the context of an uncertain problem, the elasticity of the marginal payoff function, $y(E)$, has a natural interpretation as an index of risk aversion. Hence we can say from (9) that an increase in society's risk aversion vis-à-vis the uncertainties of climatic change will lead to a decline in the rate at which emissions should fall over time. This decline in the steepness of the profile will be accompanied by a decrease in the initial emission level, producing a flatter profile with less variation in emission levels and a smaller cumulative emission through to any given date. It is clear, then, that changes in the discount rate and the index of risk aversion have very much the effects that might be expected. Figure 1 illustrates these points.

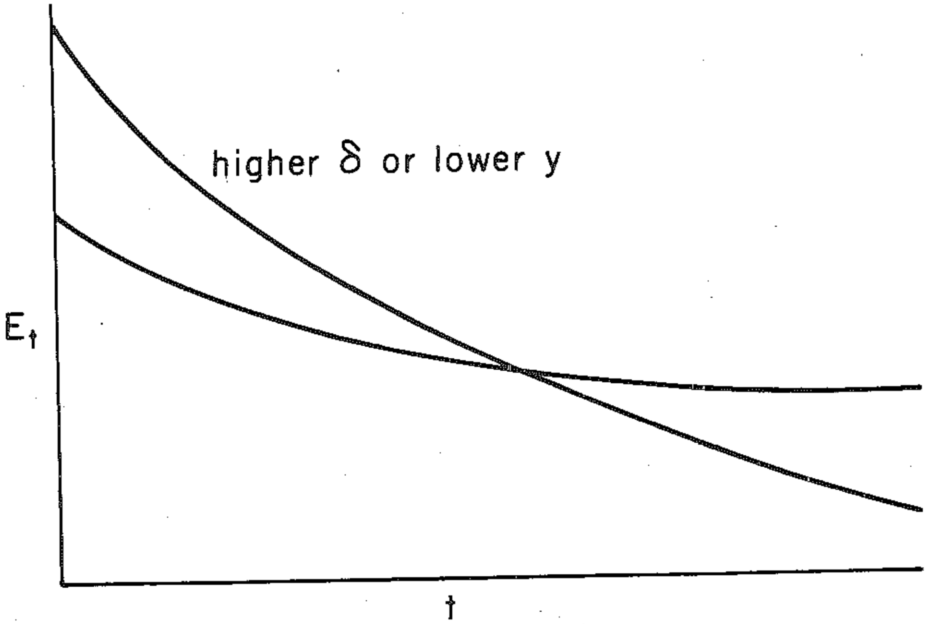

Figure 1

The role of the difference between average and marginal payoffs, $\{V' - V/E\}$, is less immediate but nevertheless quite comprehensible. To understand this, recall that the choice of any particular emission profile E amounts to a choice of a probability distribution over dates at which absorptive capacity will be exhausted. Corresponding to a particular profile, say $\hat{E}_t$, is an expected exhaustion date, say $\hat{T}$. Suppose that the level of emission just prior to $\hat{T}$ is reduced by one unit: then the loss of payoff is $V'e^{-\delta\hat{T}}$. The unit of absorptive capacity thus saved could be used to prolong the period in which this capacity remained effective; and if emission were held constant at its terminal rate, then the extra time thus gained would be $1/E_{\hat{T}}$, with the associated increase in payoff of $e^{-\delta\hat{T}}V/E$. Thus the net decrease in payoff from a marginal reduction in emissions just prior to exhaustion is

$$e^{-\delta T}\left\{V' - \frac{V}{E}\right\},$$

which explains the form of the second term in (9). For a concave function,

$(V' - V/E)$ is always negative, and the more negative it is, the greater is the decrease in payoff from a postponement of emissions. In such a situation, $(V' - V/E)$ contributes, through division by $y < 0$, a positive term to $\dot{E}/E$.

In the context of the role of the term $(V' - V/E)$, it is interesting to consider briefly the results of replacing a strictly concave $V(E)$ by a function such as that shown in Figure 2. Here $V(E) = 0$ for $E \leq \underline{E}$: for $E \geq \underline{E}$, $V(E)$ is a strictly concave function. Hence there is a threshold level $\underline{E}$ of emissions: levels of emission below this contribute nothing to the payoff. Such a function has the property that

$$V' \gtreqless \frac{V}{E}, \qquad E \lesseqgtr E^*,$$

and

$$V' = \frac{V}{E}, \qquad E = E^*.$$

Hence for a payoff function of this sort, the second term in Eq. (9) will change sign, being positive as before for $E > E^*$, but being negative for $E < E^*$. Thus the threshold effect in the payoff function reinforces the tendency for the optimal E_t to decline with time.

It is now time to summarize the conclusions of this section. Clearly the model is too simple to be anything other than a guide to thinking on this issue, but in this role it seems useful. It suggests that one essential element of our problem is that we are in danger of exhausting the unknown absorptive capacity of the atmospheric environment. How optimally to deplete an unknown absorbtive capacity is the problem that can be posed in a convenient form and that proves amenable to relatively standard analytical techniques. It can therefore be used as an element of any more general formulation of the problem of decision making in the context of economy–climate interactions.

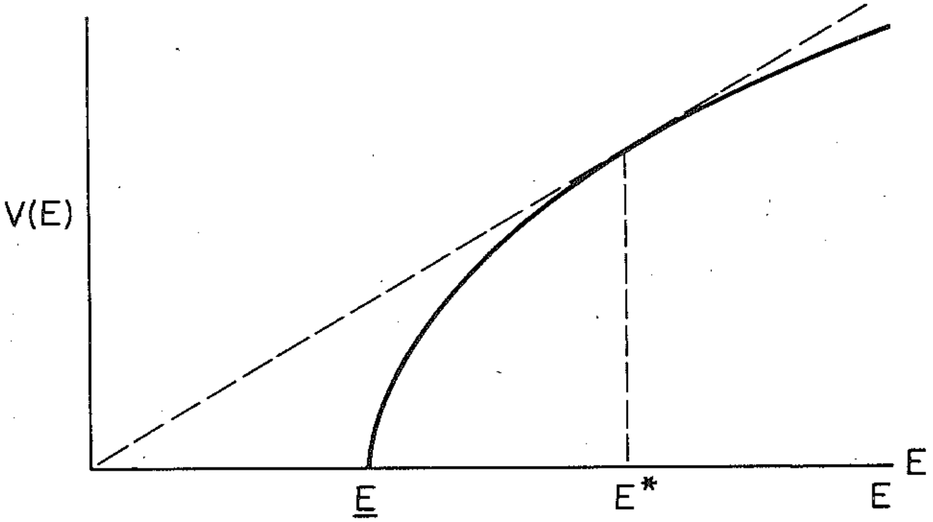

Figure 2

III. A MODEL OF RESOURCE DEPLETION AND EMISSIONS

This section addresses a slightly different formulation of the problem. It is supposed that the atmospheric environment is a common property resource which enters as an input into the production function. This environment may be in one of two states—either favorable or unfavorable to economic activity. These are denoted by $\overline{A}$ and $\underline{A}$, respectively. The environment is initially in the favorable state $\overline{A}$ but may transit stochastically into the unfavorable state $\underline{A}$: $\underline{A}$ is absorbing in the sense that once the environment is in state $\underline{A}$ it will remain there for ever. The probability of a transition from $\overline{A}$ to $\underline{A}$ is endogenous and in particular depends on cumulative emissions into the atmosphere. The source of these emissions is the use of an exhaustible resource in production: the rate of use of this at time t is R_t; and in addition to R_t and A, the remaining input to production is the capital stock, K_t. Thus production is described by

> $Y = Q(K, R, A)$, where Y is total output, $A = \overline{A}$ or $\underline{A}$, and $Q(K, R, \overline{A}) > Q(K, R, \underline{A})$ for all K and R.

We thus have a production process that depends on inputs of capital and resources, and a cooperating climatic or environmental factor. The state of the atmospheric environment depends on cumulative emissions, which in turn depends on cumulative resource use. Obviously the motivation for this model is the emission of CO_2 by the combustion of fossil fuels: this is a clear case of an emission which might change the state of the climatic environment and whose magnitude depends on cumulative use of the resource fossil fuel.

The rest of the model can be formalized as follows. The rate of emission at date t is E_t and is directly proportional to the rate of resource use R_t at time t. The constant of proportionality can be set equal to unity, and we can therefore identify E_t and R_t, and from now on will refer just to R_t. Cumulative resource depletion up to date t is denoted by Z_t:

$$Z_t = \int_0^t R_\tau \, d\tau, \qquad \dot{Z}_t = R_t.$$

The evolution of the climatic environment A is as follows:

> There exists $T > 0$ such that $A = \bar{A}$, $0 \leqslant t \leqslant T$; $A = \underline{A}$, $T < t \leqslant \infty$. Here T is a random variable whose marginal density function f has as its argument cumulative emission and depletion Z_t, thus $f = f(Z_t)$.

It follows from this formulation that the probability that the date of the change T lies in any interval (t_1, t_2) is

$$\text{Prob}(T \in (t_1, t_2)) = \int_{Z_{t_1}}^{Z_{t_2}} f(Z_t)\, dt$$

Hence, if $Z_{t_1} = Z_{t_2}$ and there is no depletion or emission in the interval (t_1, t_2), then the probability of a climatic change in that interval is zero. It is also the case that, when there is emission in the interval (t_1, t_2), the probability of a change depends not only on the level of emission in (t_1, t_2) but also on cumulative emissions prior to t_1. Although there is little formal discussion of these issues in the literature on climatic change, these seem very desirable properties.[6]

Total output Y_t may be divided between investment $\dot{K} = dK/dt$ and consumption C_t. Consumption yields value or utility at a rate given by $U(C_t)$, where $U(C_t)$ is a strictly concave function and the objective is to maximize the expected present discounted value of utility of consumption.

In addition to the production constraint, there is a constraint on total resource use steming from the fact that only a finite amount S_0 is available:

$$\int_0^\infty R_t\, dt \leqslant S_0$$

The overall problem is thus:

$$\max E \int_0^\infty U(C_t)e^{-\delta t}\, dt$$

$$\text{subject to} \int_0^\infty R_t\, dt \leqslant S_0$$

$$\dot{K} + C = Q(K, R, A).$$

The expectation operator here is over the realizations of the stochastic process governing T, the date of transition from $\bar{A}$ to $\underline{A}$.

We define

$$S_t = S_0 - \int_0^t R_\tau\, d\tau, \qquad \dot{S}_t = -R_t,$$

and also

$$W(K_T, S_T) = \max \int_T^\infty U(C_t)e^{-\delta(t-T)}\, dt,$$

where the maximization is subject to the technological conditions operative after T and the initial conditions K_T and S_T at T: $W(K_T, S_T)$ is thus a state valuation function for the state variables of the system at the transition date T. Hence we may write the maximand as

$$E\left\{\int_0^T U(C_t)e^{-\delta t}\, dt + W(K_T, S_T)e^{-\delta T}\right\}$$

$$= \int_{Z_0}^{Z_\infty} f(Z)\left\{\int_0^T U(C_t)e^{-\delta t}\, dt + W(K_T, S_T)e^{-\delta T}\right\} dZ$$

$$= \int_0^\infty f(Z)R\left\{\int_0^T U(C_t)e^{-\delta t}\, dt + W(K_T, S_T)e^{-\delta T}\right\} dT.$$

Define

$$F(Z_t) = \int_T^\infty f(Z_t)R_t\, dt$$

and recall that

$$Z_t = S_0 - S_t.$$

Then the maximization can finally be written as:

$$\max \int_0^\infty U(C_t)\, F(S_0 - S_t)e^{-\delta t}\, dt + \int_0^\infty W(K_t, S_t)\, f(S_0 - S_t)R_t e^{-\delta t}\, dt$$

$$\text{subject to } C_t + \dot{K}_t = Q(K_t, R_t, \bar{A}),$$

$$\dot{S}_t = -R_t,$$

$$\lim_{t\to\infty} S_t \geqslant 0.$$

Once again the problem has been converted into a relatively tractable dynamic optimization problem which can be solved by the maximum principle. The problem is essentially an optimal resource depletion problem, with the added complication that resource depletion generates emissions whose cumulative impact may be to trigger a climatic change which affects the production possibilities of the economic system.[7] In fact it is clear that, as everything after capital T is summarized in a state valuation function, one could equally well suppose that the climatic change alters the nature of the payoff or utility function, for example, by making the climate less (or more) pleasant.

Letting λ_t and μ_t be costate variables, the Hamiltonian is

$$H = U(C)F(S_0 - S_t)e^{-\delta t} + W(K_t, S_t)f(S_0 - S_t)R_t e^{-\delta t}$$
$$+ \lambda(Q(K, R, \bar{A}) - C)e^{-\delta t} - \mu R_t e^{-\delta t}$$

and the conditions necessary for a maximum, assuming an interior solution, are

$$U'(C)\,F(S_0 - S_t) - \lambda = 0, \tag{10}$$

$$W(K_t, S_t)\,f(S_0 - S_t) + \lambda Q_R - \mu = 0, \tag{11}$$

(where $Q_R = \partial Q/\partial R$), and

$$\dot{\lambda} - \delta\lambda = -\lambda Q_k, \tag{12}$$

$$\dot{\mu} - \delta\mu = UF' + W_s\, fR + W\, f'R. \tag{13}$$

In the case of a "pure" optimal resource depletion problem, the equivalent of (13) would be $\dot{\mu} = \delta\mu$, with the shadow price of the resource constraint rising at the rate δ. Clearly matters are a great deal more complex in the present case: note that $F' < 0$ and f' may be positive or negative, so that in turn $\dot{\mu}$ may have either sign. The possibility that the shadow price on the resource constraint may fall and even become negative is quite in accordance with intuition. The resource is of course a scarce input and for this reason should have a positive and growing shadow price. But it is also a source of emissions which clearly have a negative economic value, and this creates a tendency in the opposite direction.

Once again defining

$$q(z) = \frac{f(z)}{F(Z)}, \qquad y(c) = \frac{U''C}{U'},$$

(10) and (12) yield

$$\frac{\dot{C}}{C} = \frac{\delta}{y} + q\frac{R}{y} - \frac{Q_k}{y}. \tag{14}$$

For a pure resource depletion problem where resource use has no environmental side effects, one would derive an equation identical to (14) except that the term qR/y would be missing (precisely as when the conditional probability q is zero or the rate of emission R is zero). We can therefore regard (14) as the result for the pure resource depletion model with the discount rate augmented by an amount of qR, which depends both on the conditional probability of a climatic change and on the rate of emission.

From (11) and (13), together with the simplifying assumption that $W_s = 0$, it is possible to derive

$$\frac{1}{Q_R}\frac{d}{dt}(Q_R) = Q_k + \frac{1}{FU'Q_R}(\delta Wf + UF'). \tag{15}$$

Equation (15) is again identical to a result that emerges from the pure resource depletion model except for the presence of the second term on the right-hand side. Without this second term, it is simply a statement of the obvious efficiency condition that the rate of return to the resource R (which equals the rate of change of its price, which in turn under competitive conditions equals its marginal product Q_R) should equal that to the capital good, which is given by its marginal productivity Q_K.

In the present context, however, there are two components to the return to the resource—one connected with its productive use, and the other associated with its impact on the climate. Recalling that $F' < 0$, and noting that if $\underline{A}$ is very much worse than $\bar{A}$, it is clear that the second term on the right-hand side in (15) is likely to be negative. Hence in this case the rate of return to the productive use of the resource is to be equated to a lower value than in the pure depletion case, so that its private (as opposed to social) marginal product will rise more slowly. This in turn indicates, as one would expect, that the solution path will have a lower rate of depletion of the resource than in the case of no climatic side effects.

IV. CONCLUSIONS

The object of this paper has been to discuss decision rules in the field of economy–climate interactions, with particular emphasis on the appropriate reaction to uncertainty. The most natural reaction to a highly uncertain problem is to attempt to simplify it by invoking certainty equivalence results and replacing it by an equivalent certain problem. The properties of such an approach have been studied extensively (see, for example, Theil, 1961; Malinvaud, 1972; Henry, 1974), and it is known to be optimal in the linear-quadratic-Gaussian (LQG.) case. However, certainty equivalence results which depend on the system to be studied being LQG. are almost certainly of little use in the present area because of the very complex nonlinearities that may be present in environmental systems and because there is no evidence that a normal distribution is the most appropriate formulation of the stochastic element of the problem. My approach has therefore been to model directly the nonlinear, nonnormal aspects of the system. There are occasions when certainty equivalence results are available for dynamic nonlinear nonnormal systems, as shown in Dasgupta and Heal (1974); some such results are apparent in Section III, but they are of limited generality.

The particular phenomenon that I have been concerned to model is one of a discrete and calamitous change in environmental conditions induced by cumulative economic activity. The melting of the polar icecaps would perhaps be the stereotype of such an event. It is an open question whether such discrete and calamitous changes are the ones most likely to occur—though, by definition, they are the ones most likely to be important if they do occur. Possibly therefore their expected impact, in the statistical sense, is greatest. This is an issue that must be resolved by further work in the physical sciences. However, it is I think important to emphasize that the methods and models used here are by no means limited to the analysis of discrete and unpleasant events, although these provide a striking and simple framework for displaying their potential. The essence of the models and techniques developed here is that there is a date, random but endogenous, at which cumulative economic activity induces climate changes. One could adapt the model to the analysis of a sequence of relatively minor events or to the analysis of a gradual shift of climatic regime, the beginning date of which was uncertain. This latter case would be modeled by having the stochastic process in the model describe the date of initiation of a process of alteration of the production possibilities in the model. It seems that the only important possibility to which this framework could not be adapted is a series of very local but significant climate changes induced by global economic activities: this would seem to require a more disaggregated approach. Analysis of this type of phenomenon would also seem to require disaggregation of the global climate models now in use, in order to supply data about possible patterns of microclimate change.

There is relatively little literature to which the analysis of this paper can be related. As indicated in the previous sections, there are papers with similar analytical structures, but they are usually focused on rather different issues. Those most closely related are Cropper (1976) and Heal (1979). The former specifically considers the regulation of activities having random and possibly catastrophic environmental effects. Cropper's paper in fact contains two distinct models—one encompassing the regulation of activities having potentially catastrophic environmental effects, and the other encompassing the optimal depletion of an unknown resource stock. The second model is similar to that of Section II of this paper but gives a rather less detailed analysis of the solution paths. The first of Cropper's models is a more complex version of that in Section II above, allowing both for natural regeneration of the environment and for a nonlinear relationship between the level of economic activity and additions to the stock of emissions. This case appears to admit the possibility of stationary optimal solutions other than those on which $E = 0$, but conditions for the existence of such stationary solutions are not given. In the model of Section II above, conditions for the existence of nonzero stationary values of E would involve quite unreasonable assumptions about the functions and parameters, and this would be true even if the model were extended to include natural regeneration of the atmospheric environment or nonlinear stock-flow interactions.

Another related analysis is that of Heal (1982), which is based on the earlier work by Heal and Ryder (1973). This also analyzes the optimal control of pollution-emitting activities which have a cumulative impact, but in this case within a deterministic framework. The environment is not seen as having a fixed absorptive capacity in that model but as undergoing gradual deterioration as the pollutant stock increases. Such a model is quite different from the ones already discussed, focusing on a rather different set of issues. It is therefore of interest that its behavior is in some respects similar to that of Cropper's model, with the possibility of a multiplicity of nonzero stationary values of the pollution variable. This model also suggests that the optimal pollution policy is very sensitive to the initial conditions of the model, being quite different for different initial regimes.

It is perhaps appropriate to end this paper by indicating an important aspect of the problem that has not been discussed here. As mentioned above, the models analyzed are very aggregative and treat the system as a uniform whole. In fact there is some indication that long before major global changes in the environment occur, significant local changes could occur in some regions. Recent trends in the sub-Saharan belt provide an example of a purely regional but nonetheless very significant climatic change. In some cases these changes could be beneficial and in others harmful. This suggests that when these micro-level effects are considered there could be a considerable international divergence of interests, with some nations or regions standing to gain and others to lose from the continuation of climate-affecting activities. In all cases, of course, both the timing and the magnitude of the gains and losses would be very uncertain. Obviously considerations of this type raise questions about the scope for international agreements in this field, and it could perhaps be important to analyze this issue within a game-theoretic or bargaining context.

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NOTES

1. These issues are surveyed in detail in other papers in this section of the volume. See also International Workshop on Climate Issues (1978), Frisken (1973), and Larmuth (1979). The impact of agricultural activities on the climate is given little prominence in most reviews,

but Larmuth quotes the startling fact that in 1872 some 14 percent of potential usable land was desert whereas by 1977 this figure had risen to 55 percent. He also attributes most of the change to overintensive agricultural use.

2. Private communication, Science Policy Research Unit, University of Sussex.

3. In an earlier paper, Cropper (1976) has analyzed environmental problems in a very similar framework.

4. This formulation of the problem draws heavily on Heal (1979), which discusses inter alia the optimal depletion policy for an exhaustible resource whose initial stock is unknown. The analogy with the present problem will be immediately clear. This problem was studied earlier by Cropper (1976, Sec. III), Kemp (1976), Gilbert (1976), and Loury (1978).

5. Cropper (1976), in a related but more complex model, investigates possible stationary solutions: she can characterize them, but not guarantee their existence.

6. An analogous stochastic optimization problem is discussed in Dasgupta et al. (1976, 1980). There the same assumptions about the probabilities have a very clear grounding in the structure of the problem.

7. The problem discussed in Dasgupta and Heal (1974) could be regarded as the "pure depletion" version of the present problem. There resource depletion has no impact on the stochastic process to which the economy is subject.

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THE HEDONIC PRICE TECHNIQUE AND THE VALUE OF CLIMATE AS A RESOURCE

A. Myrick Freeman, III

I. INTRODUCTION

Human activity could lead to changes in climate at a global, regional, local level. Such climatic changes could affect human welfare through variety of channels. For example, changes in rainfall or temperature cou affect agricultural productivity. The availability and cost of raw materia could be affected. Populations could respond to changes in climate migration and the relocation of the economic activity. And there mig be direct effects on utility or welfare due to changes in the amenity tributes of climate.

There are difficult conceptual and empirical problems in predictin quantifying, and valuing these potential effects of climate change. F example, it seems likely that any economic assessment would have

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