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NOTES ON THE ECONOMIC CONSEQUENCES OF UNCERTAIN PRODUCT QUALITY*

1. Introduction

The motivation for the following exercises stems from the observation that a consumer is frequently uncertain about the quality of a product that he is considering buying, and therefore about the extent to which it will in fact render him the services that he might expect of it. The importance of this uncertainty is gaining increasing practical recognition in a number of ways. Organisations that sell consumers information about the quality of products are increasing their membership and the scope of their activities. There is an increasing volume of consumer protection legislation, which is designed to limit the range within which a manufacturer is free to vary the quality of his product. And finally, public attention is increasingly being focussed on issues of product liability and on the terms of guarantees that sellers offer with their products. Although these issues are all of obvious importance, and are clearly all amenable to analytical examination, it is nevertheless true that there is very little discussion of them in the economic literature. (Notable exceptions are the work of Akerlof [1], Brown [5] and Kihlstrom [8], and also the celebrated legal and economic analysis of Calebresi [6]). The remainder of this paper contains a number of simple models that can be used to examine the effect of uncertainty about product quality on consumer and producer behaviour.

2. Uncertain Product Quality: A One-Good Model

Suppose a consumer's satisfaction to be described by a function $U(n, q, Y-np)$ where n is the number of units of a good that he buys, q is the quality of these units (assumed to be the same for all units), Y is his income, and p the price of the product. It will be convenient to distinguish between q^*, the *true* or *actual quality* of the product, and $\hat{q}$, which will be called the *apparent quality*. The consumer is unaware of q^*, but does know

G. Schwödiauer (ed.), Equilibrium and Disequilibrium in Economic Theory, 295–314.

the value of $\hat{q}$, which he treats as an estimate of q^*. In particular, he assumes that

$$(1) \qquad q^* = \hat{q} - \theta$$

where $-1 \leqq \theta \leqq +1$, and θ is a random variable whose marginal distribution is given by $f(\theta; x)$, where x is a parameter. An increase in x will be supposed to represent a *mean-preserving spread* of the distribution of θ, and vice versa. (The concept of a mean-preserving spread is discussed in an economic context by Rothschild and Stiglitz [10]: Blackwell [3] discusses some related ideas.) This model may be given one of two institutional interpretations:

(i) there is a certain degree of random variation in the quality of the producer's output. $\hat{q}$ is the expected value of its quality, but the consumer knows from past experience that q^* may be distributed around this value.

(ii) the consumer has no direct experience of the quality of the output. Products are tested by a Consumer Advisory Bureau, and this announces that the quality of the product is $\hat{q}$. The consumer knows however that this process is subject to error: hence the quality of any products that he buys, q^*, may be distributed around $\hat{q}$.

The remainder of this section investigates, in a simple framework, the effect of a change in the degree of uncertainty about q^* on the consumer's behaviour, and on the behaviour of the supplier of the good. According to interpretation (i) above, this amounts to investigating the effect of a change in the producer's quality control policies: according to (ii), it amounts to looking at the effect of a change in the accuracy with which the Bureau carries out its product testing. Analysis of these effects may provide a basis for deciding whether such testing is desirable, and how the costs of financing it might be allocated.

Within the framework now established, the consumer's budget problem is clearly to

$$(2) \qquad \begin{array}{ll} \text{Maximise} & \int_{-1}^{+1} U(n, \hat{q} - \theta, Y - np) f(\theta; x) d\theta \\ \text{Subject to} & 0 \leqq np \leqq Y \end{array}$$

The quantity $Y - np$ can be interpreted as a surrogate for 'other goods'. On the assumption that the solution to (2) is an interior maximum (involving consumption of both the good under consideration and 'other goods'), so that the constraint in (2) is not binding, the first-order condition for a solution to (2) is

$$(3) \qquad \int_{-1}^{+1} \left(U_n(n, q^*, Y-np) - pU_y(n, q^*, Y-np)\right) f(\theta; x) d\theta = 0$$

where $U_n = \partial U/\partial n$, $U_y = \partial U/\partial y$, $y = Y - np$. Let $\tilde{n}$ be the consumer's optimal purchase, defined implicitly by (3). To investigate how $\tilde{n}$ varies as the degree of uncertainty varies, we apply the Implicit Function Rule to (3) to yield

$$(4) \qquad \frac{\partial \tilde{n}}{\partial x} = -\frac{\left[\int_{-1}^{+1} (U_n - pU_y) f_x \, \mathrm{d}\theta\right]}{\int_{-1}^{+1} (U_{nn} - 2pU_{ny} + p^2 U_{yy}) f(\theta; x) \, \mathrm{d}\theta}$$

Assume $U()$ to be a strictly concave function, so that the denominator is negative and the sign $\partial \tilde{n}/\partial x$ is equal to the sign of the integral in square brackets. For convenience let

$$(5) \qquad U_n - pU_y = \frac{\mathrm{d}U}{\mathrm{d}n}$$

Then we are interested in the sign of

$$(6) \qquad \int_{-1}^{+1} \frac{\mathrm{d}U(n, q^*, Y-np)}{\mathrm{d}n} \cdot \frac{\partial f(\theta; x)}{\partial x} \, \mathrm{d}\theta$$

Let $F(\theta : x) = \int_{-1}^{\theta} f(\Omega; x) \, \mathrm{d}\Omega$ so that $F_\theta = f(\theta; x)$.

(6) can be expressed as

$$\int_{-1}^{+1} \frac{\mathrm{d}U(n, q^*, Y-np)}{\mathrm{d}n} \frac{\partial^2 F(\theta; x)}{\partial x \, \partial \theta} \, \mathrm{d}\theta$$

which on integration by parts yields

$$\int_{-1}^{+1} \frac{\partial}{\partial q^*} \left[\frac{\mathrm{d}U}{\mathrm{d}n}\right] F_x(\theta; x) \, \mathrm{d}\theta + \left[\frac{\mathrm{d}U}{\mathrm{d}n} F_x\right]_{-1}^{+1}$$

It follows from the definition of a mean-preserving spread (see [10], pp. 229–231, especially Equations (5), (6) and (7)) that

$$F_x(-1;x) = F_x(+1;x) = 0 \tag{7}$$

$$\int_{-1}^{\theta} F_x(\Omega; X)\,\mathrm{d}\Omega \geqq 0 \tag{8}$$

$$\int_{-1}^{+1} F_x(\theta; x)\,\mathrm{d}\theta = 0 \tag{9}$$

Hence (6) equals

$$\int_{-1}^{+1} \frac{\partial}{\partial q^*}\left[\frac{\mathrm{d}U}{\mathrm{d}n}\right] F_x(\theta; x)\,\mathrm{d}\theta$$

and integrating by parts once again, applying (7), (8) and (9),

$$(6) = +\int_{-1}^{+1} \frac{\partial^2}{\partial q^{*2}}\left[\frac{\mathrm{d}U}{\mathrm{d}n}\right]\left(\int_{-1}^{\theta} F_x(\Omega;x)\,\mathrm{d}\Omega\right)\mathrm{d}\theta \tag{10}$$

If $\dfrac{\partial^2}{\partial q^{*2}}\left[\dfrac{\mathrm{d}U}{\mathrm{d}n}\right]$ is single signed over the range of permissible θ values,

$$\text{Sign}\,\frac{\partial \tilde{n}}{\partial x} = \text{Sign}\,\frac{\partial^2}{\partial q^{*2}}\left[\frac{\mathrm{d}U}{\mathrm{d}n}\right] \tag{11}$$

because of (8). This is a simple extension of a result of Rothschild and Stiglitz ([10], p. 67) which has been discussed extensively by Diamond and Stiglitz ([6], p. 4). A mean-preserving increase in uncertainty leads an increase or decrease in the consumption of the good according as the total derivative of U w.r.t.n. (i.e. taking into account effects working via the budget constraint) is convex or concave in q^*. (In the standard Arrow-Pratt [2], [9] approach to the theory of risk aversion, the index of risk-aversion is the ratio of the second and first derivatives of a function of one variable. The sign of the third derivative is then one of the factor determining whether there is increasing or decreasing risk-aversion. Possibly the presence of a third derivative in (11) could be related to a generalised notion of increasing or decreasing risk-aversion.)

From (5), it follows that

$$\text{(12)} \qquad \frac{\partial^2}{\partial q^{*2}}\left[\frac{\mathrm{d}U}{\mathrm{d}n}\right] = U_{nqq} - pU_{yqq}$$

Consider the specific case in which

$$U(n, q^*, y) = n^\alpha q^{*\beta} y^\gamma$$

Then

$$U_{nqq} - pU_{yqq} = \beta(\beta - 1)\,\tilde{n}^{\alpha-1} q^{*\beta-2} y^{\gamma-1} \left(\alpha(Y - p\tilde{n}) - p\gamma\tilde{n}\right)$$

So that, for $\beta < 1$,

$$\text{(13)} \qquad \text{Sign}\,\frac{\partial \tilde{n}}{\partial x} = \text{Sign}\left(\frac{p\tilde{n}}{Y} - \frac{\alpha}{\alpha+\gamma}\right)$$

(13) *implies that whether an increase in riskiness leads to an increase or decrease in the amount of the good purchased, depends entirely on the proportion of his income the consumer spends on it:* if this exceeds $\alpha/\alpha + \gamma$, then the number rises, and vice versa. One might rationalise this by saying that if the consumer initially chooses to spend a large fraction of his income on a good, then its services are clearly important to him. If the variability of its quality rises, then the possibility that any given purchase will yield a low service flow is increased (as is the probability that it will yield a high flow): if the services of the good are sufficiently important, or the individual sufficiently risk-averse, this may lead him to insure by buying more of it. Thus if we need the services of light bulbs, and the variability of their life rises, we may be lead to buy more bulbs because the probability of a given number failing in a given time is increased. Possibly one can detect elements of this result in the general form given in Equation (12). Introspection suggests the following pattern of signs for the derivatives of $U(\ )$:

$$U_n > 0, \quad U_{nq} > 0, \quad U_{nqq} < 0$$

$$U_y > 0, \quad U_{yq} \gtreqless 0, \quad U_{yqq} < 0$$

Thus an increase in the quality of a good increases its marginal utility, but does so at a diminishing rate: an increase in quality may raise or lower the marginal utility of income, but whatever the sign of the effect, it shows diminishing returns. Now if n is large, y is small, so that U_n is low and U_y, high. But if n is high and U_n low, it is unlikely that U_n can be increased much further by increases in q: hence U_{nq} will be low and nearly constant, with U_{nqq} near zero. In such a case, the sign of (12) is likely to equal that of $-pU_{yqq}$.

There may be another way in which the tendency to buy more of a good whose quality becomes less certain, a tendency at first sight somewhat unintuitive, can be rationalised. Suppose that the function $U(n, q, Y-np)$ takes the very simple form $U_1(nq)+U_2(Y-np)$: then a necessary condition for a maximum of expected utility is that

$$\frac{\int_{-1}^{+1} \frac{\mathrm{d}U_1}{\mathrm{d}(nq)} \cdot q \cdot f(q)\,\mathrm{d}q}{\frac{\mathrm{d}U_2}{\mathrm{d}(Y-np)}} = P$$

Taking 'other goods' as a numeraire, this says that the expected marginal utility of a good should equal its price. For a strictly concave function $U_1(\ )$, one can show that

$$\int_{-1}^{+1} \frac{\mathrm{d}U_1}{\mathrm{d}(nq)} qf(q)\,\mathrm{d}q < \bar{q} \int_{-1}^{+1} \frac{\mathrm{d}U_1}{\mathrm{d}(nq)} \quad f(q)\,\mathrm{d}q < \bar{q}\frac{\mathrm{d}U_1(n\bar{q})}{\mathrm{d}(nq)}$$

where $\bar{q} = \int_{-1}^{+1} qf(q)\,\mathrm{d}q$, so that the expected marginal utility is less than the marginal utility at the expected quality. So in order to equate the expected marginal utility to the price, it is necessary to consume more of the good than would be consumed if the quality were with certainty equal to its expected value. Of course, this argument does highlight one weakness in the analysis: in a general equilibrium situation, one would expect the price to change as the quality uncertainty changes, whereas in the present model prices are taken as parameters.

We next analyse the behaviour of the supplier of the good under consideration, who is assumed to be a monopolist. The demand for his output depends on its price, its apparent quality $\hat{q}$, and the spread of its true quality q^* about this (represented by the parameter x in the distribution $f(\theta; x)$). Total production costs depend both on output and on the true or mean quality. It is necessary at this juncture to distinguish between the two institutional interpretations mentioned earlier. Consider case (i), where the quality of the product as it leaves the factory is a random variable with mean $\hat{q}$ and actual value in any sample of q^*. In this case the producer's costs can reasonably be supposed to depend on $\hat{q}$: his profits are thus

$$P(O, \hat{q}, x)\, O - C(O, \hat{q}) \tag{14}$$

Where O represents his output level, $P(\ )$ is the inverse demand function and C the cost function. If an increase in uncertainty leads the consumer to buy more – i.e. $\partial\tilde{n}/\partial x > 0$ – then $P_x > 0$ and greater uncertainty leads to an increase in the maximum attainable profits. *Thus there may be situations where a monopolist can raise his profits by reducing the effectiveness of his quality control procedures.* (In practice, of course, quality control costs money: hence $C(\ )$ is a function of x. This reinforces the stated conclusion.) What is happening, is that the monopolist is exploiting the consumer's tendency to buy insurance, in the form of more of the good, when its quality becomes more variable. It is worth noting the contrast between this result and that reached by Akerlof [1], who found that information available to consumers about product quality could benefit manufacturers. In fact, it will be shown below that if there is another good which, in a sense to be clarified, is a substitute for the monopolist's output, then the results are more in accord with Akerlof's findings, in that an increase in uncertainty about a product's quality can never raise its sales. It is also clear that the situation might be substantially different if the producer were forced to guarantee the quality of his product: this is another issue explored later.

We next turn to be second of the two institutional interpretations mentioned earlier. There is a Consumer Advisory Bureau that tests products and announces that their tests suggest the product to have quality $\hat{q}$: consumers know the testing process to be subject to error, and recognise that the true quality q^* may be randomly distributed around $\hat{q}$:

$$q^* = \hat{q} - \theta \tag{1}$$

The producer, of course, does not know in advance what value the tester will choose for $\hat{q}$: to him, q^* is known with certainty and $\hat{q}$ is a random variable. We assume that the distribution that the producer believes $\hat{q}$

to have about q^*, is the same as that which the consumer believes q^* to have about $\hat{q}$: hence

$$\hat{q} = q^* + \theta$$

(One could justify this by supposing the tester's verdict to consists of the true quality plus noise, with the distribution of the noise known to both parties in the market.) In this formulation of the problem, the producer's profit is a random variable, depending upon θ: its expectation is

$$(15) \qquad \int_{-1}^{+1} (P(O, \hat{q}, x)\, O - C(O, q^*)) f(\theta; x)\, \mathrm{d}\theta$$

The producer will maximise this with respect to O and q^*, giving as first-order conditions

$$(16A) \qquad \int_{-1}^{+1} (P_O(O, \hat{q}, x)\, O + P - C_O(O, q^*)) f(\theta; x)\, \mathrm{d}\theta = 0$$

$$(16B) \qquad \int_{-1}^{+1} (P_q(O, q^* + \theta, x)\, O - C_q(O, q^*)) f(\theta; x)\, \mathrm{d}\theta = 0$$

If $\tilde{O}$ and $\tilde{q}$ are the profit-maximising choices defined by (16A) and (16B), then it is straightforward to investigate how these are affected by a mean-preserving increase in the variance of the tester's estimates. Only the effect of uncertainty on the choice of quality will be considered here, as this is clearly the more interesting of the two results. Clearly, if $\partial\tilde{n}/\partial x \gtrless 0$, then $P_x \gtrless 0$ – if more uncertainty leads the consumer to buy more of the good at a given price and income, then it means that a given output can be sold at a higher price. (16B) implies that

$$(17) \qquad \frac{\partial \tilde{q}}{\partial x} = \frac{\int P_{qx} O f(\theta; x)\, \mathrm{d}\theta + \int P_q O f_x(\theta; x)\, \mathrm{d}\theta}{-(\int P_{qq} O f(\theta; x)\, \mathrm{d}\theta - C_{qq})}$$

Assuming $C_q > 0$, $C_{qq} > 0$ (the cost of quality increases at an increasing rate) and $P_{qq} < 0$, it is clear that the denominator in (17) is positive. As before let

$$F(\theta; x) = \int_{-1}^{\theta} f(\Omega; x)\, \mathrm{d}\Omega \quad \text{so that} \quad F_\theta = f(\theta; x)$$

Using (7) to (9) and integrating by parts, the numerator of (17) becomes

$$(18)\qquad O\int_{-1}^{+1}\left(P_{qx}f(\theta;x)+P_{qqq}\left(\int_{-1}^{\theta}F_x(\Omega;x)\,\mathrm{d}\Omega\right)\right)\mathrm{d}\theta$$

$$\text{where}\quad \int_{-1}^{\theta}F_x(\Omega;x)\,\mathrm{d}\Omega\gtreqless 0.$$

It would not be unreasonable to assume that $P_{qx}=0$, so that the effect of uncertainty is merely to shift the demand curve in the $P-q$ plane vertically, not twisting it in any way. In such a case, the sign of $\partial\tilde{q}/\partial x$ depends entirely on whether the derivative of price with respect to quality is a concave or convex function: if concave, then an increase in uncertainty lowers the monopolist's optimal quality, and vice versa. Whether P_q will be concave or convex in q, will clearly depend upon the characteristics of the consumer's utility function, but it has not been possible to characterise this dependence in any simple way. Although the effect of greater uncertainty on the quality of the output is difficult to sign, its effect on the monopolist's expected profit is not: this effect clearly has the sign of $\partial\tilde{n}/\partial x$. Hence, in the Cobb-Douglas case at least, if the consumer bureau provides more accurate information about the quality of the monopolist's output, then this will lower his profits if the good concerned is one on which consumers spend a large fraction of their income, and vice versa.

The analysis thus far has been restricted to situations where production is monopolised. It is tempting to try and extend it to situations where there is competition among producers, but there are difficulties in doing this. They arise because an integral part of the picture of perfect competition, is that there are many sellers supplying a homogeneous commodity. It is this concept of homogeneity that causes problems when the quality of a good is a random variable. Suppose that the products of all firms have the same quality distribution. If an increase in uncertainty means that the same mean-preserving spread occurs to all of these distributions, then the consumer has no basis for discriminating between suppliers and total demand rises or falls as before: the previous analysis can be taken as applying to the equilibrium of the industry as a whole. If however alterations occur in only some of the quality distributions, then consumers will distinguish between the outputs of different firms, and the industry will be imperfectly competitive. The model of the next section is designed to throw some light on behaviour in this case.

3. Uncertain Product Quality: Competing Goods

In this section we suppose there to be two alternative goods on the market, both similar, and enquire into the effect on the consumer's purchases if the quality of one becomes more uncertain. In this case, the conditions under which an increase in the uncertainty about the quality of a good will increase its sales are considerably more restrictive, and are related to how good a substitute the other commodity is.

The model of consumer behaviour used is that suggested by Lancaster [8]: the consumer's satisfaction is a function of the amounts q_1 and q_2 of two characteristics that he consumes. Each good contains these in certain proportions, good j ($j = 1, 2$) having q_{ij} ($i = 1, 2$) units of characteristic i per unit. Hence in a deterministic situation the consumer's budget problem would be to:

$$\text{Choose } \mathrm{X} \text{ so as to maximise } U(\mathrm{Q})$$

$$\text{subject to } \mathrm{Q} = A \cdot \mathrm{X},\ \mathrm{P} \cdot \mathrm{X} \leqq Y,\ x \geqq 0.$$

where $\mathrm{Q} = (q_1, q_2)$, $\mathrm{X} = (x_1, x_2)$ is the vector giving the amounts of the goods consumed, A is a matrix with typical element a_{ij}, P is the price vector and Y the consumer's income. The quality of good 1 is assumed to be uncertain, so that a_{11} and a_{21} are random variables: to simplify matters, it is assumed that the ratio $a_{21}/a_{11} = b$ is known, so that it is only the value of a_{11} that has to be discovered to characterise good 1 fully. (The consumer thus knows the proportions in which good 1 embodies the characteristics, but does not know the absolute level of its perform ance). Letting

$$\mathrm{B} = \begin{pmatrix} 1 \\ b \end{pmatrix}, \quad \mathrm{A} = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}, \quad P = P_1, \quad P_2 = 1,$$

the consumer's problem can be stated as

$$\text{Maximise} \int U\big(a_{11} x_1 \mathrm{B} + (Y - p x_1)\mathrm{A}\big) f(a_{11}; S)\, \mathrm{d}a_{11}$$

where $f(a_{11})$ is the marginal distribution of a_{11}, and S a parameter representing its spread. The first-order condition for a maximum is

$$\int f(a_{11}; S)\,[a_{11}\mathrm{B} - p\mathrm{A}] \cdot [U_1, U_2]\, \mathrm{d}a_{11} = 0 \tag{19}$$

where $U_i = \partial U(q_1, q_2)/\partial q_i$. The Implicit Function Rule gives, if $\tilde{x}$ is the solution to (19),

$$\frac{\partial \tilde{x}}{\partial S} = \frac{\int f_S[a_{11}\mathrm{B} - p\mathrm{A}] \cdot [U_1, U_2]\, \mathrm{d}a_{11}}{-\int f \cdot [a_{11}\mathrm{B} - p\mathrm{A}] \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} [a_{11}\mathrm{B} - p\mathrm{A}]'\mathrm{d}a_{11}} \tag{20}$$

where a prime denotes the transpose of a vector. The denominator is clearly positive for strictly concave U, so the sign of $\partial \tilde{x}/S$ depends on that of the numerator, which can be written

$$\int f_s(a_{11};S) \cdot \frac{\mathrm{d}U}{\mathrm{d}x}(\ )\,\mathrm{d}a_{11} \tag{21}$$

where $\mathrm{d}U/\mathrm{d}x$ is the derivative of U w.r.t. x evaluated after taking into account the budget constraint. Letting

$$\int^{a} f(a_{11};S)\,\mathrm{d}a_{11} = F(a,S) \text{ so } f(a_{11};S) = F_a(a_{11};S)$$

and

$$T(a_{11};S) = \int^{a_{11}} F_S(a;S)\,\mathrm{d}a,$$

interpreting an increase in S as a mean-preserving spread, using the properties defined in (7) to (9), and integrating by parts twice,

$$(21) = \int T(a_{11};S)\frac{\partial^2}{\partial a_{11}^2}\left[\frac{\mathrm{d}U}{\mathrm{d}x}\right]\mathrm{d}a_{11} \tag{22}$$

where $T(a_{11};S) \geqq 0$ by (8). Hence the sign of $\partial \tilde{x}/\partial S$ is given by the sign of

$$\frac{\partial^2}{\partial a_{11}^2}\left[\frac{\mathrm{d}U}{\mathrm{d}x}\right] = D$$

if this is single-signed. Extensive and tedious manipulations lead to the conclusion that

$$\begin{aligned} D = {} & \frac{2}{x_1}(x_1, x_1 b)\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}\begin{pmatrix} x_1 \\ x_1 b \end{pmatrix} \\ & + (a_{11} - pa_{12})(x_1, x_1 b)\begin{bmatrix} U_{111} & U_{112} \\ U_{121} & U_{122} \end{bmatrix}\begin{pmatrix} x_1 \\ x_1 b \end{pmatrix} \\ & + (a_{21} - pa_{22})(x_1, x_1 b)\begin{bmatrix} U_{211} & U_{212} \\ U_{221} & U_{222} \end{bmatrix}\begin{pmatrix} x_1 \\ x_1 b \end{pmatrix} \end{aligned} \tag{23}$$

Fortunately it is easy to determine the signs of the components of this expression. If one unit of income is spent on good 1, this yields a_{11}/p_1 units of characteristic 1 and a_{21}/p_1 units of characteristics 2: the same unit spent on good 2 yields a_{12}/p_2 and a_{22}/p_2 units of the characteristics. Hence

$$a_{11} - pa_{12} \gtreqless 0$$

according as good 1 is better than (worse than) good 2 as a source of characteristic 1. A similar interpretation holds for $(a_{21} - pa_{22})$. Now if both of these terms were positive (negative) for all values of the random variables a_{11} and a_{21}, it is clear that the consumer would never wish to buy good 2 (good 1). Hence for at least some realisations of the random variables, $(a_{11} - pa_{12})$ and $(a_{21} - pa_{22})$ must differ in sign if the problem is to be interesting (i.e. one good must have an advantage in providing one quality, and vice versa). It is clear that the first term in the expression for D is negative for strictly concave U: the remaining quadratic forms are positive or negative according as U_1 and U_2 are convex or concave functions. In order to discuss the sign of the whole expression, it is helpful to consider the following diagram. Amounts of the two characteristics are plotted along the axes, and the amounts per unit of expenditure contained in the two goods are shown respectively by rays 0I and 0II. Good 2 offers the certain combination given by point C: good 1 may offer any combination on 0I, but it is clear from our earlier discussion that for the problem to be interesting this combination must with positive probability lie in the segment AB. At A, $a_{11} - pa_{12} = 0$ and $a_{21} - pa_{22} < 0$ at B, $a_{11} - pa_{12} > 0$ and $a_{21} - pa_{22} = 0$. By considering the special cases when the variation in the quality of good 1 is confined to small neighbourhoods of the points A and B, one can make the following statements: if

(24A) U is strictly concave
U_1 is strictly concave
the variation in good 1
is near B
then $\dfrac{\partial \tilde{x}}{\partial S} < 0$.

(24B) U is strictly concave
U_2 is strictly convex
the variation in good 2
is near A
then $\dfrac{\partial \tilde{x}}{\partial S} < 0$.

U is strictly concave
U_1 is strictly convex
the variation in good 1 then $\dfrac{\partial \tilde{x}}{\partial S} > 0.$
is near B, and X_1
is large enough

U is strictly concave
U_2 is strictly concave
(24D) the variation in good 1 then $\dfrac{\partial \tilde{x}}{\partial S} > 0.$
is near A and X_1
is large enough

It is clear that if the characteristics embodied in good 1 are distributed *only* in the interval AB, then

$$a_{11} - pa_{12} \geqq 0, \quad a_{21} - pa_{22} \leqq 0.$$

Hence one can assert, for example, that if

U is strictly concave
U_1 is strictly convex
(24E) U_2 is strictly concave then $\dfrac{\partial \tilde{x}}{\partial S} > 0.$
X_1 is large enough

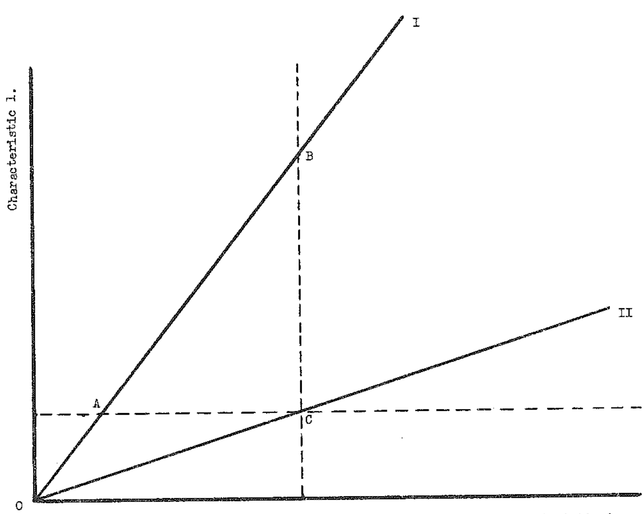

Fig. 1.

Recall that in the one-good case, it was established for a Cobb-Douglas utility function that an increase in uncertainty about its quality could lead a consumer to buy more of a good if he initially spent a lot of his income on it (see Equation (13)). There is some element of this result present in the more complex case just considered, for the first term in the expression for D is multiplied by X_1^{-1}, and this is also the only term that is negative under all plausible assumptions. Hence we can only get simple conditions under which $\partial\tilde{X}/\partial S$ is positive if X_1 is large and this first term is dominated by others that may be positive: however, that X_1 is large is not strictly speaking either necessary or sufficient for the result in question, but just makes it 'more likely'.

To establish more precise results about $\partial\tilde{X}/\partial S$ it is necessary to make more specific assumptions about the utility function. To this effect, we suppose it to be Cobb-Douglas again, with

$$U(q_1, q_2) = q_1^{\alpha} q_2^{\beta}$$

One can show, again after tedious manipulations, that in this case

$$\begin{aligned} D = {} & X_1 \alpha(\alpha - 1)\, q_1^{\alpha-2} q_2^{\beta-1} a_{22}(\alpha X_2 - \beta P X_1) + \\ & + 4X_1 b U_{12} + 2x_1 b^2 U_{22} + a_{21} X_1^2 U_{21} \frac{(\alpha - 1)}{q_1} \\ & - p a_{12} X_1^2 \frac{U_{11}}{q_1} (\alpha - 2) + \\ & + (a_{11} - p a_{12}) \left(\frac{2X^2 b U_{11}}{q_2} + \frac{X^2 b^2 U_{12}}{q_2} (\beta - 1) \right) \\ & + (a_{21} - p a_{22}) \left(\frac{2X^2 b U_{21}}{q_2} (\beta - 1) + \frac{X^2 b^2 U_{22}}{q_2} (\beta - 2) \right) \end{aligned}$$

In fact the first term in this unpleasant expression is similar to the term in Section 1 from which Equation (13) is derived: the sign of $(\alpha X_2 - \beta p X_1)$ is of course the same as the sign of $(\alpha p_2 X_2 - \beta p_1 X_1)$ which in turn equals

$$\text{sign} \left(\frac{\alpha}{\beta} - \frac{P_1 X_1}{P_2 X_2} \right)$$

Hence the first term in D is positive or negative according as

$$(26) \qquad \frac{P_1 X_1}{P_2 X_2} \gtrless \frac{\alpha}{\beta},$$

a result very similar to that in (13). When will it be possible to move from this limited statement to one about the sign of D as a whole? Fortunately there is a simple answer to this query: suppose the matrix A to be diagonal, so that

$$(27) \qquad a_{12} = a_{21} = b = 0.$$

Then in equation (25), all terms other than the first are identically zero, and the sign of $\partial \tilde{X}/\partial S$ depends only on the proportions in which the consumer divides his income between the two goods, as given in inequality (26). Assumption (27) corresponds to a situation where each good supplies only one characteristic, and each characteristic is provided by only one good. In this case the goods are not in a technical sense substitutes for each other: if the consumer regards it as essential to consume both characteristics (and with a Cobb-Douglas function he clearly does), then it is essential that he should consume a positive amount of each good. In such a case, it seems in keeping with the results of section I that an increase in uncertainty about the quality of a good may lead a consumer to purchase more if it is 'important' to him that he should consume 'a lot' of the characteristic it provides. As one allows the off-diagonal elements of A to become non-zero, the simplicity of this result disappears. Of course, provided that the off-diagonal elements are sufficiently small, the first term in (25) will dominate: the effects of greater uncertainty depends on the distribution of expenditure between the goods if they are "poor" substitutes for each other in a technical sense, but becomes less readily predictable as they become better substitutes.

4. Uncertain Product Quality and the Effect of a Guarantee

If a product is known, or widely believed, to be of uncertain quality, then it is common to find that the seller of this product is prepared to offer some guarantee of its quality: the effects of such a guarantee have been have been alluded to above. In this section we examine the impact of a guarantee on the phenomena noted in Section 2 – that is, on the consumer's reaction to greater uncertainty and on the producer's incentive to improve quality control. The model is that used in Section 2: the consumer's preferences are represented by a function $U(n, q^*, Y-np)$ where q^* is again the true quality of the n units of the good that he buys. $\hat{q}$ is the mean or expected quality, and

$$q^* = \hat{q} - \theta$$

Where θ is a random variable distributed between -1 and $+1$ as $f(\theta; x)$. The guarantee takes the following form. The manufacturer picks a quality level q_0: if $q^* \geqq q_0$, he offers no compensation. If $q^* < q_0$, he contracts to offer such compensation to the consumer as will give him a fixed constant g times the satisfaction that he would have attained had the true quality been q_0. Formally, if $\tilde{n}$ is the number of units the consumer chooses to buy,

$$(28) \qquad U = \begin{matrix} U(\tilde{n}, q^*, Y-\tilde{n}p) & \text{if} \quad q^* \geqq q_0 \\ g \cdot U(\tilde{n}, q_0, Y-\tilde{n}p) & \text{if} \quad q^* < q_0 \end{matrix}$$

Thus if $g = 1$, the guarantee takes the form of an assurance that, whatever happens, the consumer will be no worse off than if the units of the product that he bought all performed at level q_0. For $g < 1$, he is offered some fraction of this assurance, and vice versa. The consumer's expected utility from n units is

$$\int_{\theta_0}^{1} U(n, q^*, Y-np) f(\theta;x)\,\mathrm{d}\theta \; + g \cdot U(n, q_0, Y-np) \int_{+1}^{\theta_0} f(\theta;x)\,\mathrm{d}\theta.$$

where $q^0 = \hat{q} - \theta_0$,

and his optimal purchase $\tilde{n}$ satisfies

$$\int_{\theta_0}^{-1} (U_n - pU_y) f(\theta;x)\,\mathrm{d}\theta + \; g\,(U_n(q_0) - pU_y(q_0)) \int_{+1}^{\theta_0} f(\theta;x)\,\mathrm{d}\theta = 0$$

Now the sign of $\partial\tilde{n}/\partial x$ will equal the sign of

$$(29) \quad \int_{\theta_0}^{-1} (U_n - pU_y) f_x(\theta;x)\,\mathrm{d}\theta + g\,(U_n(q_0) - pU_y(q_0)) \int_{+1}^{\theta_0} f_x(\theta;x)\,\mathrm{d}\theta$$

Defining $F(\theta;x)$ in the standard way and integrating by parts, letting

$$\frac{dU}{dn} = U_n - pU_y,$$

the first term in (29) can be written as

$$\int_{\theta_0}^{-1} \frac{\partial}{\partial q^*}\left[\frac{dU}{dn}\right] F_x(\theta;x)\, d\theta - \left[\frac{dU}{dn} F_x\right] \theta_0$$

Integrating by parts once again, expression (29) takes the form

$$\int_{\theta_0}^{-1} \frac{\partial^2}{\partial q^{*2}}\left[\frac{dU}{dn}\right] \left(\int_{-1}^{\theta} F_x(\Omega;x)\, d\Omega\right) d\theta -$$

(30) $$- \left[\frac{\partial}{\partial q}\frac{dU}{dn}\int_{-1}^{\theta} F_x(\Omega;x)\, d\Omega\right]_{\theta_0}^{-1}$$

$$+ (g-1)\left[\frac{dU}{dn} F_x\right]_{\theta_0}^{-1}$$

We know from the definition of a mean-preserving spread that

$$\int_{\theta}^{+1} F_x\, d\Omega \geqq 0 \quad \text{for all } \theta,$$

$$\int_{-1}^{+1} F_x\, d\Omega = 0,$$

$$F_x(-1;x) = F_x(+1;x) = 0.$$

Hence F_x changes from positive to negative as θ falls, and is zero at at least one θ value. If we were to choose these to include θ_0, the final term in (30) would be zero. By using the Cobb-Douglas utility function used in Section 2, it is possible to establish the sign of (30). The first term of (30) is of course identical with Equation (10) of Section (2), and its sign is therefore, as in Equation (13), given by

$$\text{Sign}\left(\frac{p\tilde{n}}{Y} - \frac{\alpha}{\alpha+\gamma}\right)$$

The second term in (30) can be rewritten as

$$B\tilde{n}^{\alpha-1}q_0^{\beta-1}y^{\gamma-1}[\tilde{n}p(\alpha+\gamma)-\alpha Y]\int_{-1}^{\theta}F_x(\Omega;x)\,\mathrm{d}\Omega$$

and its sign is again equal to

$$\operatorname{Sign}\left(\frac{p\tilde{n}}{Y}-\frac{\alpha}{\alpha+\gamma}\right)$$

Finally, the third term in (30) takes the form

$$(g-1)\,\tilde{n}^{\alpha-1}q_0^{\beta}y^{\gamma-1}[\alpha Y-\tilde{n}p\,(\alpha+r)]\,F_x(\theta_0;x)$$

and its sign is

$$\operatorname{Sign}(1-g)\left(\frac{\tilde{n}p}{Y}-\frac{\alpha}{\alpha+\gamma}\right)F_x(\theta_0;x).$$

Recall that for high values of θ_0 – i.e. low values of q_0, the guaranteed level – F_x is positive. Consider this case first as it seems the more realistic. Then for $g \leq 1$, the sign of $\partial\tilde{n}/\partial x$ is the same as it was in section two with no guarantee, and it depends simply on the proportion of his income the consumer spends on the product. If $g > 1$, it is *possible* – but not necessary – that the sign of $\partial\tilde{n}/\partial x$ is reversed by comparison with the no-guarantee case. This is not really surprising: if $g > 1$, the consumer is being overcompensated for failure of the product. If the extent of overcompensation is sufficiently great, he may be tempted to bet on this over compensation: and the probability that $q \leq q_0$, so that the bet is successful, increases with an increase in the spread of the distribution. If the guaranteed level q_0 is in the upper part of the distribution, then F_x is negative. In this case some of the above effects are reversed, but this seems in empirical terms an unlikely situation.

Before leaving the present very simple model of a guarantee and its effects, it is worth enquiring briefly into its impact on the firm's decisions about quality control. Using the first institutional interpretation of Section 2, where $\hat{q}$ is the mean quality of the firm's output and q^* its actual quality, the firm's expected profit is given by

$$\pi = p(O,\hat{q},x)\cdot O - C(O,\hat{q}) - \int_{\theta_0}^{+1} g(q^*)f(\theta;x)\mathrm{d}\theta$$

which is identical to Equation (14) except for the final stochastic term. The function $g(q^*)$ represents the monetary payments the firm has to make to the consumer if the true quality q^* is less than q_0. Clearly

$$\frac{\partial\pi}{\partial x} = P_x \cdot O - \int_{\theta}^{+1} g(\hat{q}-\theta) f_x(\theta;x)\, \mathrm{d}\theta =$$

$$= P_x \cdot O + \int_{\theta_0}^{+1} g'(\hat{q}-\theta)\, F_x(\theta;x)\, \mathrm{d}\theta + g(q_0)\, F_x(\theta_0;x).$$

after defining $F(\theta) = \int_{\theta}^{+1} f(\Omega;x)\, \mathrm{d}\Omega$ and integrating by parts. Suppose $g(q_0) = 0$: then the third term vanishes. Clearly $g' = \mathrm{d}g/\mathrm{d}q^* < 0$, and in the range in which we are interested $F_x > 0$. Hence the effect of introducing a guarantee is to make $\partial\pi/\partial x$ more negative than otherwise, making it less likely than otherwise that the monopolist will wish to reduce the tightness of his quality control.

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NOTE

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