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Discounting by committee☆

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ABSTRACT

We study a dynamic social choice problem in which a sequence of committees must decide how to consume a public asset. A committee convened at time t decides on consumption at t, accounting for the behaviour of future committees. Committee members disagree about the appropriate value of the pure rate of time preference, but must nevertheless reach a decision. If each committee aggregates its members' preferences in a utilitarian manner, the collective preferences of successive committees will be time inconsistent, and they will implement inefficient consumption plans. If however committees decide on the level of consumption by a majoritarian vote in each period, they may improve on the consumption plans implemented by utilitarian committees. Using a simple model, we show that this occurs in empirically plausible cases. Application to the problem of choosing the social discount rate is discussed.

1. Introduction

Suppose that a society needs to decide on an intertemporal consumption plan for some public asset. A committee is convened at each point in time, and tasked with determining how much to consume in the current period. The members of each committee have differing opinions about the pure rate of social time preference (PRSTP), or utility discount rate, that should be applied to this problem. Some favour a high discount rate, while others believe that different time periods should be treated more equally, and thus favour a low discount rate. Moreover, the current committee knows that future consumption choices will also be made by committees exhibiting similar disagreements on discount rates. How should such committees proceed, given the heterogeneity in opinions on discount rates?

Although it may seem abstract, this question is inspired by an important practical problem in public economics: how should governments discount future utilities when evaluating public policy decisions? The appropriate normative value of the PRSTP has been debated at least since Ramsey's (1928) seminal work on optimal national savings. Subsequent commentators have argued the merits of a variety of values for the PRSTP without a clear 'best' value emerging, and different governments have adopted different values for public decision-making. The social time preferences economists prescribe for public decision-making today are still highly heterogeneous (Arrow et al., 2013). This has been highlighted by the long-standing debate about the appropriate value of the PRSTP for the evaluation of climate change policies (Nordhaus, 2008; Stern, 2007). A recent survey of experts on social discounting (Drupp et al., forthcoming) shows significant variation in their prescriptions for the PRSTP (see Fig. 2).

Given the persistent normative disagreements about the PRSTP, it is natural to ask whether methods from social choice theory can be used to obtain a compromise between opposing viewpoints. In this paper, we examine perhaps the most common such methods: utilitarian aggregation and majoritarian voting. Under the utilitarian approach, committees seek to maximize a weighted sum of the time preferences advocated by their members in each period, while under majoritarian voting, committee members vote on the current level of consumption, and a Condorcet winner (if it exists) is implemented.

The utilitarian approach is appealing, as Jackson and Yariv (2015) have shown that any social choice rule that is non-dictatorial

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(i.e. sensitive to the preferences of more than one individual) and respects unanimity (roughly, if everyone prefers consumption stream **C** to **C**′ then **C** is socially preferred to **C**′) is equivalent to utilitarianism in the setting we study. However, while no-dictatorship and unanimity are compelling properties in isolation, they lead to a time inconsistency problem when combined with another assumption: time invariance (i.e. preferences over future consumption streams are identical in all time periods). Millner and Heal (2018) have argued that while time invariance is an excessively strong assumption in intra-group intertemporal decision problems (e.g. allocation between family members), it is plausible when modeling inter-group choices like those facing the successive committees studied in this paper. Thus, if a utilitarian approach to resolving disagreements is adopted, the collective preferences of successive committees will conflict with one another. Rational utilitarian committees will anticipate the actions of future committees, and react optimally to them, inducing a dynamic game between committees. The equilibrium of this game will be seen as inefficient by every committee.

The inefficiency of the consumption path implemented by utilitarian committees means that it is possible that voting could give rise to superior outcomes. If each committee holds a majoritarian vote on the level of current consumption, and members of the current committee rationally anticipate the outcome of future votes, we show that the equilibrium consumption path under voting will correspond to the optimal plan of the median member. Further analysis shows that a majority of committee members will prefer this voting equilibrium to the utilitarian equilibrium, regardless of the choice of aggregation weights in the utilitarian objective function. We extend this result to welfare comparisons, finding conditions on the distribution of PRSTPs under which the voting equilibrium is superior to the utilitarian equilibrium according to utilitarian committees' own objective functions. Using survey data on economists' recommended values for the PRSTP, we show that these conditions are often satisfied in practice. There is thus a sense in which voting may be 'self-stable' (Barbera and Jackson, 2004) relative to utilitarianism: a majoritarian vote between voting and utilitarian aggregation of PRSTPs will always lead to voting being adopted as the aggregation method. By contrast, a utilitarian comparison of voting and utilitarian equilibria will often favour voting.

The paper is structured as follows. We discuss related literature next, before developing our simple model of dynamic public choice with disagreements about the PRSTP in Section 2. This section contains the bulk of our analysis. We first derive the equilibrium behaviour of utilitarian committees, and show that they choose inefficient consumption paths. Next, we derive the equilibrium behaviour of committees that vote on consumption. Finally, we contrast these two preference aggregation methods, deriving results on committee members' ordinal preferences between the implemented equilibria, and comparing them from the perspective of utilitarian committees' own collective preferences. Section 3 discusses the results, and draws some lessons for the choice of the PRSTP in social discounting formulae.

1.1. Related literature

The literature on aggregation of opinions about social discount rates stems from the work of Weitzman (1998, 2001), who focuses on aggregation of expert opinions on real (i.e. consumption) discount rates, rather than pure time preferences. Weitzman takes a sample of opinions as to the appropriate (constant) real discount rate for project evaluation, treats these as uncertain estimates of the 'true' underlying rate, and takes expectations of the associated discount factors to derive a declining term structure for the 'certainty equivalent' real discount rate. As Freeman and Groom (2015) observe, opinions about real discount rates conflate ethical views about welfare parameters such as the PRSTP with empirical estimates of consumption growth rates — they mix tastes and beliefs (see Dasgupta, 2001, pp. 187–190 and Gollier, 2016 for further discussions of Weitzman's approach). This suggests that it is important to pursue approaches that treat preference aggregation as a distinct problem. Our work highlights difficulties that may arise in practice when decision-makers with a distribution of ethical views attempt to form consensus social preferences, and contrasts the equilibrium outcomes that arise from standard preference aggregation methods.

The possibility that utilitarian preference aggregation could lead to time inconsistency when agents favour different values of the PRSTP has been noted by several authors (Marglin, 1963; Feldstein, 1964; Jackson and Yariv, 2015). Millner and Heal (2018) argue that, while this is not a generic feature of utilitarianism as an normative theory (see also e.g. Hammond, 1996), as a positive matter it is likely to occur when distinct groups of agents are tasked with decision-making in each time period, as occurs in the setting we study here. Our work thus falls somewhere on the boundary between normative and positive analysis: we study *positive* properties of the equilibrium consumption choices that would be implemented by sequences of committees that seek to aggregate their members' *normative* views on social time preferences. Alternative approaches to the aggregation of time preferences are pursued by Gollier and Zeckhauser (2005), Jouini et al. (2010), and Millner (2018).

2. The model

We focus on a sequential social choice problem in which a sequence of committees, each composed of $N > 1$ members indexed by $i = 1 \ldots N$, must choose how to consume a public asset. For the sake of analytical convenience, we assume that N is odd, and that time is continuous. Each committee exists for a single moment in time, and controls the value of consumption in that moment alone. Committee members are drawn from a stable population at each moment, and their tenure lasts for only that moment. The distribution of members' opinions on the PRSTP is assumed to be independent of time.[1]

The public asset committees must manage is modeled as a risk-free asset S that yields a constant (net) rate of return $r \geq 0$. If the asset is consumed at rate C_τ at time τ, the dynamics of S are given by

$$\dot{S} = rS_\tau - C_\tau \tag{1}$$

where $\dot{S} = dS/d\tau$, and the initial value of S at time τ_0 is S_0. This simple model has many possible interpretations. For example, S could be a stock of environmental quality, a publicly owned natural resource, or the value of a country's sovereign wealth fund.

Member i in a committee constituted at time τ is assumed to have discounted utilitarian preferences over future consumption streams denoted by $V_{i\tau}$, with a PRSTP $\delta_i > 0$:

$$V_{i\tau} = \int_\tau^\infty U_i(C_t)e^{-\delta_i(t-\tau)}dt. \tag{2}$$

Committee members have heterogeneous opinions on the appropriate value of the PRSTP, i.e. there exist indices i,j such that $\delta_i \neq \delta_j$. We will interpret δ_i as i's *normative* opinion on the appropriate rate of *social* impatience. Thus, the preferences (Eq. (2)) do *not* represent members' private preferences over their own consumption, but rather

[1] If N is reasonably large, this is a mild assumption, as sampling variation in members' preferred values of the PRSTP will be small. All the results below can be easily extended to the case of a continuum of committee members (i.e. zero sampling variation) by taking the limit as $N \to \infty$.

reflect their normative opinions on social preferences. These opinions could arise from first-principles ethical reasoning (as in e.g. Ramsey, 1928; Arrow, 1999; Stern, 2007), or from a revealed preference approach that identifies social wellbeing with the satisfaction of consumers' preferences (as in e.g. Nordhaus, 2008). We take no stance on which method for specifying the PRSTP is 'correct' in this paper. Indeed, the motivation for examining social choice rules is that we believe that all the methods deployed in the literature have strengths and weaknesses, and reasonable people can have legitimate disagreements on how to proceed.

To focus the analysis on the heterogeneity in views on the PRSTP, we assume that all members favour the same iso-elastic utility function:

$$U_i(C) = U(C) = \begin{cases} \frac{C^{1-\eta}}{1-\eta} & \eta \neq 1 \\ \ln C & \eta = 1, \end{cases} \tag{3}$$

where $\eta > 0$ is the elasticity of marginal utility. This assumption is clearly restrictive, but is necessary to make analytical headway. We discuss the dependence of our results on this assumption in the Conclusions. The special case $\eta = 1$ will turn out to be especially useful, as only in this case can all the equilibria we study be solved analytically.

In what follows, it will sometimes be useful to use shorthand notation, which we collect here for convenience. We write the weighted average of the elements of a vector $\vec{x} = (x_1, \ldots, x_N)$, taken with weights $\vec{y} = (y_1, \ldots, y_N)$ $(y_i \geq 0, \sum_{j=1}^N y_j = 1)$ as the vector dot product

$$\vec{x} \cdot \vec{y} := \sum_{i=1}^{N} x_i y_i. \tag{4}$$

In addition, the weighted average of the n-th power of the elements of $\vec{x}$ will be written as

$$(\vec{x})^n \cdot \vec{y} := \sum_{i=1}^{N} (x_i)^n y_i. \tag{5}$$

Thus, exponents are understood to act element-wise on vectors. Finally, the unweighted average of the elements of a vector $\vec{x}$ will be denoted by

$$\langle \vec{x} \rangle := \frac{1}{N} \sum_{i=1}^{N} x_i. \tag{6}$$

2.1. Utilitarian aggregation

In this subsection, we describe the consumption choices that will be made if committees make decisions by aggregating the preferences of their members in a utilitarian manner. That is, we suppose that a committee at time τ adopts a social choice rule that can be represented by a function W_τ, where

$$W_\tau = \sum_i y_i V_{i\tau}, \tag{7}$$

and the weights y_i satisfy $y_i \geq 0$, $\sum_i y_i = 1$.[2] Proposition 1 in Jackson and Yariv (2015) shows that when committee members have discounted utilitarian preferences (Eq. (2)) and a common utility function, a social choice rule respects unanimity[3] if and only if it is of the form Eq. (7). If in addition we require social preferences to be non-dictatorial, we must have $y_i > 0$ for at least two values of i. Thus, utilitarian aggregation embodies two very basic desirable properties of preference aggregation in this setting. In the remainder of the paper, we assume that $y_i > 0$ for all i, so that the committee's collective preferences are sensitive to each member's favoured discount rate.

An immediate consequence of the heterogeneity of opinions on discount rates and the utilitarian preferences (Eq. (7)) is that committees that are constituted at different times will not agree on the ranking of consumption streams, i.e. their preferences will be inconsistent. To see the intuition for this, observe that a committee at time τ_1 with preferences (Eq. (7)) would like to assign weight $y_i e^{-\delta_i(\tau_2 - \tau_1)}$ to member i's views at time $\tau_2 > \tau_1$. However, from the perspective of the committee formed at τ_2, the appropriate weight on opinion i at time τ_2 is just y_i. Hence, the time inconsistency problem. Indeed, classical results (e.g. Strotz, 1955) show that the preferences (Eq. (7)) are time consistent if and only if all the members agree on the value of δ, or the aggregation weights y_i assign zero weight to all but one of the members' views (see also Millner and Heal, 2018).

Outside of these degenerate cases, we can solve for the consumption choices of rational utilitarian committees by treating the problem as a dynamic game. That is, a committee at τ_1 rationally anticipates the consumption decisions of future committees at all $\tau > \tau_1$. The committee at τ_1 then makes the best decision it can, taking the choices of future committees as given. This induces a dynamic game between committees, and we look for sub-game perfect equilibria of this game as in e.g. Phelps and Pollak (1968) and Laibson (1997). Infinite horizon dynamic games of this kind admit many sub-game perfect equilibria in general (Laibson, 1994; Krusell and Smith, 2003). We use a standard equilibrium selection method to single out a unique equilibrium. We interpret our infinite horizon model as the limit of a finite horizon model as the horizon length tends to infinity.[4] Under this interpretation, the equilibrium, if it exists, can be shown to be unique, and to correspond to a linear Markov Perfect Equilibrium (MPE) (see e.g. Laibson, 1994; Krusell et al., 2002).[5] When the equilibrium exists under this procedure, we will refer to it as the 'limit equilibrium'.

To determine the limit equilibrium, we must find the linear MPE of the dynamic game between committees. A Markovian strategy in our context is a function $\sigma(S)$ such that consumption at time t is given by $C_t = \sigma(S_t)$ for all $t \geq \tau$. A strategy $\sigma(S)$ is an MPE if, in the limit as $\epsilon \to 0$, when committees at times $t \in [\tau + \epsilon, \infty)$ in the future use the rule $\sigma(S)$, the best response of the current committee in $t \in [\tau, \tau + \epsilon)$

[2] Note that we have assumed that only the social preferences advocated by committee members at time τ are deemed relevant to the aggregation exercise conducted by committee τ. Thus, the current committee accounts for the views of its members, but does not account for the judgements of past or future committees. All intertemporal concerns are captured by current decision-makers' views on social time preferences. These assumptions seem to us to capture the problem of choosing a PRSTP by committee in practice. Future decision-makers have no voice in current debates on social impatience, so the future is only relevant to the extent that current decision-makers are concerned about it. See Millner and Heal (2018) for further discussion of this assumption.

[3] The formulation of this property adopted by Jackson and Yariv (2015) is: Social preferences $\succeq_S$ satisfy unanimity iff for all consumption streams $\mathbf{C}, \mathbf{C}'$, a) $\forall i, \mathbf{C} \succeq_i \mathbf{C}' \Rightarrow \mathbf{C} \succeq_S \mathbf{C}'$, and b) $\forall i, \mathbf{C} \succ_i \mathbf{C}' \Rightarrow \mathbf{C} \succ_S \mathbf{C}'$.

[4] In a model with finite horizon T, committee member i at time τ's preferences are: $\tilde{V}_{i\tau} = \int_\tau^T U(C_t) e^{-\delta_i(t-\tau)} dt$. The infinite horizon limit corresponds to the $T \to \infty$ limit of the equilibrium of the dynamic game when T is finite.

[5] To clarify, there is always a unique equilibrium of the finite horizon game, but the limit of this equilibrium as the horizon length tends to infinity may not exist, as welfare may become unbounded in this limit.

is also to use $\sigma(S)$. An MPE is linear if the equilibrium strategy is of the form $C_t = \sigma(S_t) = AS_t$ for some $A > 0$.

The next proposition characterizes the limit equilibrium of the game between committees:

Proposition 1.

1. *Assume that the elasticity of marginal utility $\eta \geq 1$. Then, the limit equilibrium of the game between committees exists and is given by a consumption rule $C_t = \sigma(S_t) = AS_t$, where $A > 0$ is the unique solution of*

$$\sum_i y_i \frac{A}{\delta_i + (\eta - 1)(r - A)} = 1 \tag{8}$$

that satisfies

$$A < r + \frac{\min_i \delta_i}{\eta - 1}. \tag{9}$$

When $\eta = 1$, A may be determined explicitly:

$$A = \left[(\vec{\delta})^{-1} \cdot \vec{y}\right]^{-1}. \tag{10}$$

When $\eta < 1$, A must satisfy Eq. (8) and $A > r - \frac{\min_i \delta_i}{1-\eta}$. In this case, there may be no linear MPE.

2. *If it exists, the limit equilibrium consumption path is observationally equivalent to the optimal path according to an agent with preferences (Eq. (2)), and PRSTP*

$$\hat{\delta} = r + \eta(A - r). \tag{11}$$

3. *Member i at time τ believes that equilibrium welfare is*

$$V_{i\tau} = \begin{cases} \frac{(S_\tau A)^{1-\eta}}{1-\eta} \frac{1}{(r-A)(\eta-1)+\delta_i} & \eta \neq 1 \\ \frac{1}{\delta_i} \ln(S_\tau A) + \frac{1}{\delta_i^2}(r - A) & \eta = 1 \end{cases} \tag{12}$$

in the limit equilibrium.

Proof. See Appendix A. □

The possible non-existence of a linear MPE (and hence a limit equilibrium) when $\eta < 1$ is a well known feature of models like ours (see e.g. Phelps and Pollak, 1968). To avoid existence problems, we assume that $\eta \geq 1$ in the remainder of the paper.[6]

From the perspective of a committee in any period τ, the equilibrium described by Proposition 1 is inefficient. That is, there exist feasible consumption paths that would increase its welfare measure W_τ. However, owing to the time inconsistency of committees' preferences, these paths are not implementable. Any future committee at time $\tau' > \tau$ can increase its welfare measure by deviating from the time τ committee's optimal plan. The time τ committee knows this, anticipates the behaviour of all future committees, and reacts optimally to this knowledge. Since all committees behave this way, the resulting equilibrium is inefficient, but fully rational. Thus, although each committee's welfare measure aggregates its members' preferences efficiently, the interactions between successive committees lead to an inefficient intertemporal equilibrium.

The observational equivalence between the utilitarian equilibrium and the optimal path of a single discounted utilitarian agent with discount rate $\hat{\delta}$ allows us to use $\hat{\delta}$ as a summary statistic that captures how committees behave in equilibrium. Consider the case $\eta = 1$, in which case we have

$$\hat{\delta} = \left[(\vec{\delta})^{-1} \cdot \vec{y}\right]^{-1}. \tag{13}$$

The latter expression is a weighted harmonic mean of members' preferred discount rates. To get a very rough understanding of where this formula comes from notice that the value of a constant utility stream to a utilitarian committee is:

$$\sum_i y_i \int_\tau^\infty \bar{U} e^{-\delta_i(t-\tau)} dt = \bar{U} \sum_i y_i \delta_i^{-1} = \bar{U}(\vec{\delta})^{-1} \cdot \vec{y}.$$

On the other hand, for a single agent with discount rate $\hat{\delta}$, the value of this utility stream is

$$\int_\tau^\infty \bar{U} e^{-\hat{\delta}(t-\tau)} dt = \bar{U} \frac{1}{\hat{\delta}}.$$

Thus, we see that when $\hat{\delta}$ is chosen in accordance with Eq. (13) the utilitarian committee perceives the same 'present value of time' as a single agent with discount rate $\hat{\delta}$.

The expression for $\hat{\delta}$ is independent of the rate of return r when $\eta = 1$, as income and substitution effects from a change in r exactly cancel out in this case. When $\eta > 1$ however, this is no longer the case. In Appendix B, we prove that:

Proposition 2. *If the elasticity of marginal utility $\eta > 1$, $\frac{\partial \hat{\delta}}{\partial r} > 0$.*

This result should be interpreted with care. It does not simply say that utilitarian committees become more impatient as r increases, and thus consume more today. While this is true, this just reflects the fact that the income effect of a change in r dominates the substitution effect when $\eta > 1$, a well known result. Proposition 2 however says something stronger. Recall that $\hat{\delta}$ is the discount rate of a single agent who solves the same intertemporal decision problem as the utilitarian committees — this agent is thus also subject to the income and substitution effects that arise from a change in r. Proposition 2 says that in order to match the optimal path of a single agent to the equilibrium implemented by utilitarian committees, we need to choose an agent with a larger and larger discount rate as r increases. Thus, even netting out the effect of a change in r on the single agent's optimal path, we still need to increase his discount rate if we are to match the behaviour of the utilitarian committees. This suggests that the substitution effect of a change in r is weaker and/or the income effect is stronger for utilitarian committees than it is for single agents. Since the single agent and the committees face the same intertemporal budget constraint by construction, it must be that the substitution effect is weaker. Indeed, this conforms to intuition. The fact that the current utilitarian committee cannot commit future committees to implement the consumption plan it would like, means that it should rationally reduce its estimate of the benefits from savings, since it knows that future committees will act

[6] Note that an optimal path in the single agent version of the model only exists if $\delta + (\eta - 1)r > 0$, where δ is the agent's utility discount rate (see part 2 of Appendix A). This condition is always satisfied if $\eta \geq 1$, but may fail if $\eta < 1$. Thus, non-existence of an equilibrium is not unique to the dynamic game that arises when preferences are heterogeneous, but also occurs in a standard optimal control problem with a single agent.

sub-optimally from its perspective. Thus, an increase in r has less effect on a utilitarian committee's ability to substitute consumption between current and future periods than it does for a single agent who faces no commitment problems.

2.2. Voting

The fact that utilitarian committees choose inefficient consumption plans in equilibrium suggests that alternative methods for aggregating member's opinions could improve on utilitarian preference aggregation. The most natural alternative to consider is majoritarian voting. Aside from being widely deployed in practice, majority rule has been shown to satisfy desirable properties of preference aggregation over a larger domain of preferences than any other ordinal social choice rule (Dasgupta and Maskin, 2008, see also May, 1952). Yet as Sen (2017, p. xxvii) observes, 'when it comes to welfare economics, majority decision is not a particularly just, or even plausible, way of judging alternatives'. Sen is referring here to the fact that decisions implemented by majority rule will generally not promote more comprehensive measures of social welfare. Indeed, while majoritarian ballots have been a staple of the positive theory of public choice for decades (e.g. Black, 1948; Downs, 1957; Meltzer and Richard, 1981), they are seldom invoked as a means to pursue normative social objectives. We will show below however that our model provides one instance in which majoritarian voting may be desirable according to such a normative objective. Voting on consumption may lead all committees to achieve higher levels of utilitarian welfare W_τ than attempting to maximize W_τ directly.

Suppose that consumption C_τ is to be decided by ballot in each period. In each period τ, each committee member may nominate a single value of C_τ. All members vote over each pair of nominated consumption values, and the value that gets a majority of votes wins each pairwise contest. A Condorcet winner (if it exists) is a value of C_τ that wins every pairwise contest. If there is a Condorcet winner, it is implemented.

Since the current choice of C_τ influences the consumption choices that will be made in future ballots, committee members must anticipate the outcomes of those ballots when forming their preferences over current consumption C_τ. We assume that members are rational, and thus anticipate the outcomes of all future ballots when forming their preferences over C_τ.[7] The following proposition characterizes the equilibrium consumption path that emerges from this sequence of ballots:

Proposition 3. *Committee members at time τ who anticipate the outcome of votes over public consumption in future periods have single-peaked preferences over current consumption C_τ. Thus, the equilibrium of a majoritarian voting model with ballots in every period is the optimal consumption plan of the median member.*

Proof. See Appendix C. □

This result may seem to be in conflict with the analysis of voting over consumption streams in Jackson and Yariv (2015). They show that voting over consumption streams in unrestricted domains is generically intransitive, and thus voting equilibria cannot be represented by the preferences of a single individual such as the median agent. Their analysis assumes however that votes are once off, whereas in our result ballots are repeated, so that in each period members are only voting over a single, unconstrained, value of consumption. The repeated ballot formulation is compelling, as it does not require the somewhat far-fetched assumption that members believe that a consumption plan that is decided on today will automatically be implemented by all future committees.

[7] As in the analysis of the MPE in Proposition 1, we assume a finite horizon voting model, and take the limit as the horizon length tends to infinity.

2.3. Utilitarian aggregation vs. voting

We are now in a position to compare the equilibrium implemented by utilitarian committees to that implemented by committees that vote on consumption.

Our first result provides a lower bound on the number of committee members who prefer the voting equilibrium to the equilibrium implemented by utilitarian committees:

Proposition 4. *In every period, a majority of members prefer the voting equilibrium to the equilibrium implemented by any utilitarian committee.*

Proof. See Appendix D. □

The heart of the proof of this result is to show that committee members have single-peaked preferences over dynamic consumption plans that are optimal for *some* discounted utilitarian agent. Once we know this, the result follows from the fact that the equilibrium implemented by a utilitarian committee is observationally equivalent to the optimal plan of an agent with discount rate $\hat{\delta}$ (see Proposition 1), and the fact that the plan corresponding to the median agent's discount rate is a Condorcet winner.

This result is independent of any assumptions about utilitarian committees' aggregation weights. Thus, regardless of *how* such committees aggregate preferences, a majority of members will think that voting on discount rates will lead to superior outcomes than attempting to maximize W_τ directly. Next, we take this observation further by investigating when voting dominates direct maximization of W_τ, according to the welfare measure W_τ itself.

We begin with a result that does not require any assumptions about the model's primitives (i.e. $\vec{\delta}, \vec{y}, r$, and η). Let δ^* be the discount rate associated with a utilitarian committee's most preferred single agent optimal plan (assuming that this quantity is well defined — we will prove this below). That is, if C_t^δ is the optimal consumption path according to a committee member with discount rate δ, δ^* is given by

$$\delta^* = \arg\max_{\delta} \sum_i y_i \int_\tau^\infty U(C_t^\delta) e^{-\delta_i(t-\tau)} dt.$$

In addition, denote the median committee member's discount rate by δ_m.

Proposition 5. *Assume that $\eta \geq 1$. Then,*

1. *Utilitarian committees have single-peaked preferences over single agent optimal plans.*
2. *δ^* exists, is unique, and is independent of τ. Moreover,*

$$\delta^* < \hat{\delta},$$

where $\hat{\delta}$ is defined in Eq. (11).

3. *Voting yields higher utilitarian welfare than the equilibrium implemented by utilitarian committees if:*

$$\delta^* < \delta_m < \hat{\delta}. \quad (14)$$

The first part of the proposition shows that utilitarian committees inherit the single-peakedness of their members' preferences on the space of single-agent optimal paths. Although it is not true in general that a weighted average of single-peaked preferences is single-peaked, this does hold for our model. Since utilitarian committees' preferences over discount rates are single-peaked, it is useful to know something about where their 'bliss point' discount rate lies. The second part of the proposition shows that it always lies *below* the discount rate that replicates the utilitarian equilibrium. This is intuitive, as the commitment problem utilitarian committees face always causes them to be more short-termist than they would like to be. The final part of the proposition combines parts 1 and 2 to provide a simple sufficient (but not necessary) condition for voting to dominate the utilitarian equilibrium. This condition applies regardless of the model's primitives, although checking whether Eq. (14) is satisfied requires us to specify these primitives.

In order to progress beyond this general result, we must make additional assumptions. We begin by examining the case $\eta = 1$, where analytic results are possible, thus enabling clean comparative statics on the distribution of opinions $\vec{\delta}$. For $\eta > 1$, we must resort to numerical methods.

2.3.1. Logarithmic utility function (i.e., $\boldsymbol{\eta}$=1)

When η=1 the equilibrium condition for the dynamic game between utilitarian committees has a closed form solution (Eq. (10)), and members' opinions on welfare in the voting and utilitarian equilibria can be computed analytically. This allows us to obtain a sharp result on when voting will dominate utilitarian aggregation, according to a utilitarian objective function.

Proposition 6. *Assume $\eta = 1$. Then voting gives rise to higher utilitarian welfare than direct attempts to optimize W_τ if and only if*

$$\frac{\delta^*}{\delta_m}\ln\left(\frac{\hat{\delta}}{\delta_m}\right) < \frac{\hat{\delta}}{\delta_m} - 1. \tag{15}$$

where

$$\delta^* = \frac{\vec{\delta}^{-1}\cdot\vec{y}}{\vec{\delta}^{-2}\cdot\vec{y}}$$
$$\hat{\delta} = (\vec{\delta}^{-1}\cdot\vec{y})^{-1}.$$

Proof. See Appendix F. □

In order to understand the constraints the conditions (14) and (15) place on the distribution of discount rates in more detail, we specialise to a natural choice for the vector of aggregation weights $\vec{y}$. Since members only differ in their attitudes to time, a natural comparability requirement is that if the consumption path C_t does *not* depend on time, all members should contribute equally to the committee's utilitarian welfare measure W_τ. There is a unique choice of aggregation weights that ensures this: we must pick the weight y_i on member i with discount rate δ_i to be[8]:

$$y_i = \frac{\delta_i}{\sum_j \delta_j}. \tag{16}$$

With this choice for $\vec{y}$, some simple algebraic manipulations show that the sufficient condition in Eq. (14) becomes

$$\langle\delta^{-1}\rangle^{-1} < \delta_m < \langle\delta\rangle, \tag{17}$$

and the necessary and sufficient condition in Eq. (15) becomes

$$\frac{\langle\delta^{-1}\rangle^{-1}}{\delta_m}\ln\left(\frac{\langle\delta\rangle}{\delta_m}\right) < \frac{\langle\delta\rangle}{\delta_m} - 1. \tag{18}$$

Thus, in this case, the ranking of utilitarian aggregation and voting depends on the arithmetic mean $\langle\delta\rangle$, the harmonic mean $\langle\delta^{-1}\rangle^{-1}$, and the median δ_m.

We illustrate the implications of Eqs. (17)–(18) in an example in which there are three members with discount rates $\delta_1 \le \delta_2 \le \delta_3$. Define $h_{12} := \delta_1/\delta_2, h_{13} := \delta_1/\delta_3$. Clearly, $0 \le h_{12} \le h_{13} \le 1$. Simple calculations show that Eq. (17) is equivalent to

$$\frac{1}{2}(1 + h_{13}) < h_{12} < \frac{2h_{13}}{1 + h_{13}}. \tag{17a}$$

Similarly, Eq. (18) is equivalent to

$$\frac{3h_{12}}{1 + h_{12} + h_{13}}\ln\left(\frac{1}{3}(1 + h_{12}(1 + \frac{1}{h_{13}}))\right) < \frac{1}{3}\left(1 + h_{12}\left(1 + \frac{1}{h_{13}}\right)\right). \tag{18a}$$

This inequality can be solved numerically to determine the constraints it places on h_{12} for given h_{13}.

Fig. 1 plots the set of three element distributions that satisfy these conditions. The figure shows that Eq. (18a) is *almost* equivalent to requiring that the distribution of discount rates have positive non-parametric skewness, i.e. $\langle\delta\rangle > \delta_m$. The vast majority of distributions that satisfy Eq. (18a) have this property. However, there is a small set of distributions that satisfy Eq. (18a), but have $\langle\delta\rangle < \delta_m$, indicated by region *C* in the figure. The figure also demonstrates that the condition (17a), satisfied in region *B* of the figure, is sufficient but by no means necessary for voting to dominate the equilibrium implemented by utilitarian committees.

The approximate positive skewness condition needed for voting to dominate the utilitarian equilibrium in this example conforms to intuition. When the distribution of discount rates exhibits positive skewness there is a long tail of large discount rates above the median. These discount rates have a disproportionate influence on the equilibrium implemented by utilitarian committees. If a committee with aggregation weights (Eq. (16)) could control consumption for all time, it would choose a consumption path that satisfies impatient members in the short run, and patient members in the long run. Short-run consumption choices are thus always dominated by the concerns of impatient committee members. However, because the current committee cannot bind the hands of future committees, its consumption choice is in effect always short-termist, and thus impatient members' preferences exert a large influence on it. Since this is true for every committee, the equilibrium consumption path implemented by a sequence of utilitarian committees will be biased towards more impatient members. This is reflected in the fact that the equilibrium implemented in this case is observationally equivalent to the optimal plan of an agent with discount rate $\langle\delta\rangle$. It is well known that the arithmetic mean is sensitive to the large 'outliers' that exist when the distribution of δ is positively skewed. By contrast, the voting equilibrium is robust to the presence of these large outliers, and is thus not subject to the same distortions.

[8] Equivalently, we could have defined member i's welfare measure as $\tilde{V}_{i\tau} = \delta_i\int_\tau^\infty U(C_t)e^{-\delta_i(t-\tau)}dt$, in which case the choice of aggregation weights in Eq. (16) corresponds to assigning equal weight to each $\tilde{V}_{i\tau}$ in W_τ.

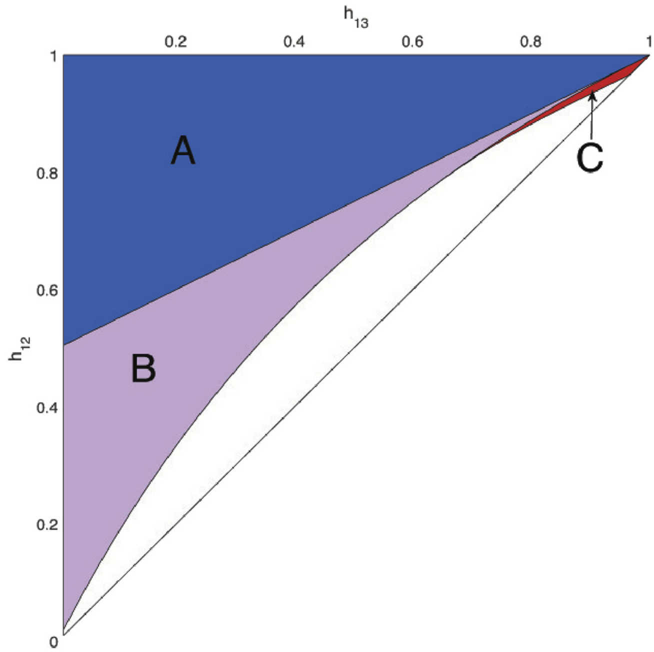


Fig. 1. When does voting dominate the equilibrium implemented by utilitarian committees? The figure represents all three element distributions of discount rates with elements $\delta_1 \le \delta_2 \le \delta_3$, using the ratios $h_{12} = \delta_1/\delta_2$, and $h_{13} = \delta_1/\delta_3$, where $h_{12} \ge h_{13}$. Aggregation weights are given by Eq. (16). Voting dominates utilitarian aggregation in the region $A \cup B \cup C$. Condition (17), i.e. $\langle\delta^{-1}\rangle^{-1} < \delta_m < \langle\delta\rangle$, is satisfied in region B, while $A \cup B$ is the region where $\langle\delta\rangle > \delta_m$. Region C is the small set of distributions for which Eq. (18) is satisfied, but $\langle\delta\rangle < \delta_m$.

2.3.2. More general iso-elastic utility functions (i.e., $\boldsymbol{\eta > 1}$)

When $\eta > 1$, the equilibrium condition (10) must be solved numerically. In order to do this, we must specify the distribution of opinions on the PRSTP. We will use a distribution of discount rate prescriptions elicited from economists who are experts in public project evaluation (Drupp et al., forthcoming). This distribution is illustrated in Fig. 2. In addition, throughout this subsection we assume that the utilitarian committee aggregates preferences using the welfare weights in Eq. (16), i.e. $y_i \propto \delta_i$. Given these assumptions, standard numerical methods can be used to solve Eq. (10), and the

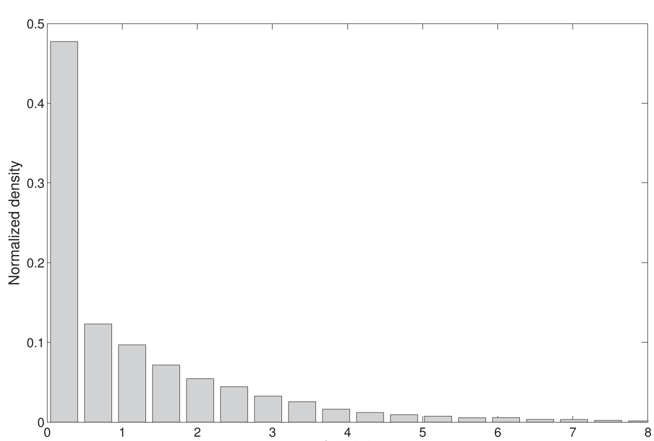


Fig. 2. Distribution of the recommended value of the pure rate of social time preference δ for public project appraisal, from the Drupp et al. (forthcoming) survey of economists. 180 responses were recorded in the original sample. A kernel density fit has been applied to smooth out the dataset.

welfare committees achieve under voting and utilitarian aggregation can be computed as functions of the parameters r, η.

To quantify the welfare differences between the two preference aggregation methods, it is helpful to have a method for converting these differences into consumption units. We will make use of the stationary equivalent of a welfare value W (Weitzman, 1976), defined as the constant consumption value $\bar{C}$ that satisfies

$$\sum_i y_i \int_\tau^\infty U(\bar{C}) e^{-\delta_i (t-\tau)} dt = W. \tag{19}$$

When $U(C)$ is iso-elastic with $\eta \neq 1$, the percentage change Δ between the stationary equivalent under voting ($\bar{C}_V$) and utilitarian aggregation ($\bar{C}_{UA}$) takes a simple form:

$$\Delta = 100 \left[\frac{\bar{C}_V - \bar{C}_{UA}}{\bar{C}_{UA}} \right] = 100 \left[\left(\frac{W_V}{W_{UA}} \right)^{\frac{1}{1-\eta}} - 1 \right], \tag{20}$$

where W_V is the welfare achieved under voting, and W_{UA} the welfare achieved under utilitarian aggregation.

We begin by observing that for the distribution of discount rates in Fig. 2, we have:

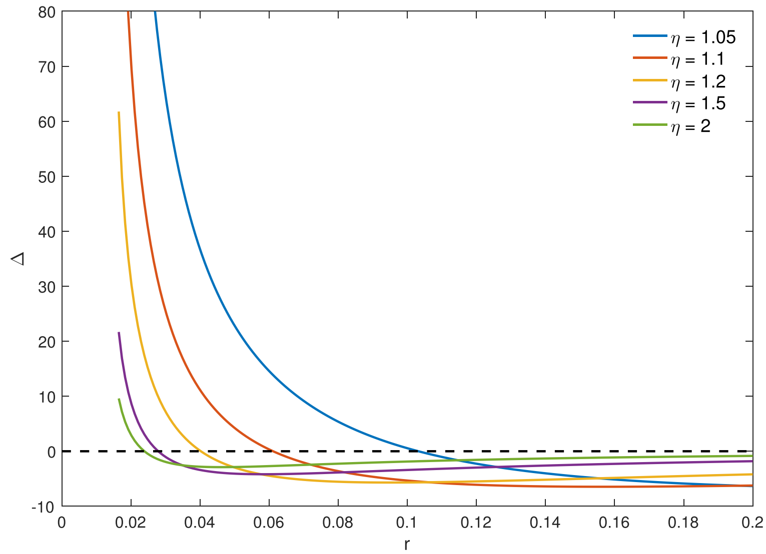
$$\delta^* = \langle\delta^{-1}\rangle^{-1} = 1.18 \times 10^{-5}\%, \quad \delta_m = 0.53\%, \quad \hat{\delta} = \langle\delta\rangle = 1.15\%.$$

Thus, the condition in Eq. (17) is satisfied, and we know that voting dominates utilitarian aggregation when $\eta = 1$, for all values of r. To see whether this result extends to other values of η, r, consider Fig. 3(a), which plots Δ as a function of r for several values of η. The figure shows that for any fixed value of η, voting is preferred to utilitarian aggregation for lower values of r. For r smaller than about 2%/yr, voting yields very substantial benefits for all values of $\eta > 1$ (so large that we have cut off the graph to preserve readability). However, as r increases, the balance tips in favour of the utilitarian aggregation method. The region of the (r, η) parameter space where each preference aggregation method dominates is depicted in Fig. 3(b).

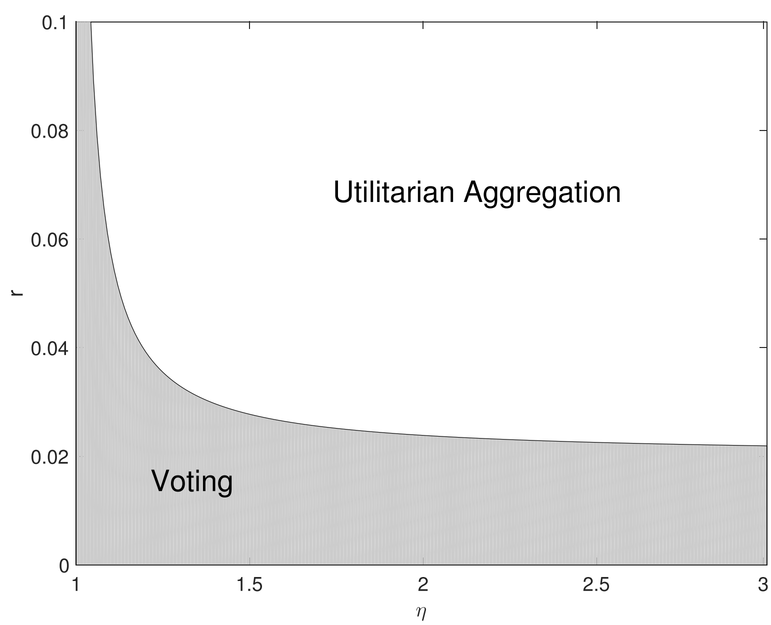
The results in Fig. 3(a) and (b) are difficult to explain intuitively, as we need to know how both equilibria, and the way they are evaluated, vary with r and η. Since we do not have an analytic expression for the consumption path implemented by utilitarian committees, this is a difficult task. Ultimately, welfare comparisons for $\eta > 1$ depend on the empirical details, including the distribution of discount rates. Nevertheless, Fig. 3(b) shows that in our calibration of the model there is a large region of empirically plausible parameter values where voting yields better utilitarian outcomes than the equilibrium utilitarian committees would choose for themselves.

3. Conclusions

In this paper, we have contrasted two of the most natural preference aggregation methods that committees of decision-makers might employ when attempting to resolve normative disagreements about social time preferences: utilitarianism, and majoritarian voting. While in normal circumstances voting cannot hope to compete with direct optimization of a utilitarian objective function, the time inconsistency of utilitarian committees' collective preferences leads them to implement inefficient consumption plans. We have shown that this can cause voting to yield better outcomes, according to utilitarian committees' own objectives. Indeed, our simple empirical analysis using an elicited distribution of expert opinions on the pure



(a) Dependence of Δ, the percentage difference between stationary equivalents under voting and utilitarian aggregation, on η and r.



(b) The shaded area depicts the region of parameter space where voting dominates utilitarian aggregation, according to utilitarian welfare measure.

Fig. 3. Voting vs. utilitarian aggregation.

rate of social time preference suggests that this occurs in empirically plausible cases.

An interesting feature of model is that, regardless of how utilitarian committees aggregate preferences, a majority of decision-makers will believe society to be better off if committees vote on consumption than if they attempt to maximize their utilitarian welfare measure directly. There is thus a sense in which majoritarian voting is 'self-stable' with respect to utilitarian aggregation. The concept of self-stability was introduced by Barbera and Jackson (2004) in the context of their study of constitutions. A voting rule X for 'ordinary business' (e.g. deciding on discount rates) is self-stable relative to an alternative voting rule Y if when society votes on whether to change the voting rule from X to Y, and uses the rule X to adjudicate this vote, it chooses to stick to X.[9] With a little modification, we can adapt this concept to our analysis. If majority voting is also used to decide whether to use majoritarian voting or utilitarianism to aggregate members' preferences, voting will always be selected. By contrast, if the choice between the two aggregation methods is made based on

[9] The definition of self-stability in Barbera and Jackson (2004) extends to more than just pairwise comparisons of decision rules, we have simplified it to fit our setting.

comparisons of the equilibrium utilitarian welfare they achieve, we have shown that there are plausible circumstances under which voting dominates utilitarianism. In these circumstances, majoritarian voting is self-stable, but maximizing utilitarian objectives is not.

We believe that our results have practical implications for resolving disagreements about the welfare parameters that enter social discounting formulae. The social discount rate r_t is the rate of decline of the marginal rate of substitution between future consumption at time t and current consumption:

$$r_t = -\frac{1}{t}\ln\frac{U'(C_t)e^{-\delta t}}{U'(C_0)}.$$

If public decision-making is efficient, costs and benefits that are realized at future times t must be discounted using the discount rate r_t (see e.g. Gollier, 2012).

It is clear from the expression above that the PRSTP δ is a critical input to r_t — small changes in its value can have a very large impact on the evaluation of public projects with long-term consequences (see e.g. Heal and Millner, 2014). In practice, governments revise their choices of the social discount at semi-regular intervals (Gollier and Hammitt, 2014), and debates about the appropriate value of the PRSTP are invariably part of this process. While the processes that are used to resolve ethical disagreements about the PRSTP are currently *ad hoc* and rather opaque, our work takes a more systematic approach to the aggregation of viewpoints on social impatience. Our conclusion is that voting on the PRSTP is likely to have advantages over utilitarian aggregation in practice. Our simple empirical analysis suggests that a consensus value of $\delta \approx 0.5\%$/yr could emerge from such a vote. This value is considerably smaller than that advocated by e.g. Nordhaus (2008) in his analysis of climate change policy (he favours 1.5%/yr), but larger than the value of zero advocated by e.g. Stern (2007) and Gollier (2012) based on their personal ethical views.

The main limitation of our analysis is the assumption that committee members share a common utility function. This assumption is made only for technical reasons. It is possible to extend our results on the voting equilibrium to the case where committee members favour different iso-elastic utility functions.[10] However, solving for the equilibrium of the dynamic game between utilitarian committees becomes difficult in this case, as the committee's preferences are no longer iso-elastic, and thus the limit equilibrium is generically non-linear. Similar tractability issues arise for more general (i.e. non-linear) production functions. Nevertheless, the qualitative finding that voting may dominate direct utilitarian aggregation will continue to hold even in these substantially more complex cases.

Appendix A. Proof of Proposition 1

1. Let the consumption policy function be $C_t = \sigma(S_t) = AS_t$. Suppose that committees from $t \in [\tau+\epsilon, \infty)$ follow strategy $\sigma(S)$. The committee at τ's welfare from this is

$$V(C_\tau, \epsilon, A) = \int_{\tau+\epsilon}^{\infty} U(AS_t)\beta(t)dt \tag{21}$$

where

$$U(C) = \frac{C^{1-\eta}}{1-\eta};\ \beta(t) \equiv \sum_{i=1}^{N} y_i e^{-\delta_i(t-\tau)}. \tag{22}$$

We will not handle the case $\eta = 1$ explicitly in this derivation, as the correct equilibrium conditions for log utility can be obtained by sending $\eta \to 1$ in the equilibrium condition we will obtain for $\eta \neq 1$. S_t is the solution of the differential equation

$$\dot{S} = rS - AS;\ S(\tau+\epsilon) = S_\epsilon \tag{23}$$

$$\Rightarrow S_t = S_\epsilon e^{(r-A)(t-(\tau+\epsilon))}, \tag{24}$$

and S_ϵ is the stock of S the current committee bequeaths to the next committee at $t = \tau + \epsilon$. Using the state equation, and assuming that ϵ is small, we find

$$S_\epsilon \approx S_\tau(1 + \epsilon(r - C_\tau/S_\tau)). \tag{25}$$

From now on, the proof makes liberal use of several such approximations, all of which become exact in the limit as $\epsilon \to 0$. A straightforward calculation then shows that

$$V(C_\tau, \epsilon, A) = \begin{cases} \infty & \text{if } \eta < 1 \text{ and } A \le r - \frac{\min_i \delta_i}{1-\eta} \\ -\infty & \text{if } \eta > 1 \text{ and } A \ge r + \frac{\min_i \delta_i}{\eta-1} \\ \frac{1}{1-\eta}(AS_\epsilon)^{1-\eta}\sum_i y_i \frac{e^{-[\delta_i+(\eta-1)(r-A)]\epsilon}}{\delta_i+(\eta-1)(r-A)} & \text{otherwise} \end{cases} \tag{26}$$

If the current committee believes that the strategy followed by all future committees leads to unboundedly large negative (positive) welfare, it will react by sending its own consumption to zero (infinity). Strategies that lead to infinite welfare integrals thus cannot be part of an equilibrium. Thus, the equilibrium value of A must satisfy

$$\begin{aligned} A &> r - \frac{\min_i \delta_i}{1-\eta} \quad \text{if } \eta < 1 \\ A &< r + \frac{\min_i \delta_i}{\eta-1} \quad \text{if } \eta > 1. \end{aligned} \tag{27}$$

Assuming that these conditions are satisfied, we have

$$V(C_\tau, \epsilon, A) = \frac{1}{1-\eta}(AS_\epsilon)^{1-\eta}\sum_i y_i \frac{e^{-[\delta_i+(\eta-1)(r-A)]\epsilon}}{\delta_i+(\eta-1)(r-A)}.$$

The current committee's total welfare is

$$\begin{aligned} &\int_\tau^\infty U(C_t)\beta(t)dt \\ &= \int_\tau^{\tau+\epsilon} U(C_t)\beta(t)dt + \int_{\tau+\epsilon}^\infty U(C_t)\beta(t)dt \\ &\approx \epsilon U(C_\tau) + V(C_\tau, \epsilon, A) \end{aligned}$$

where the approximation becomes exact as $\epsilon \to 0$. We wish to solve for current committee's optimal choice of C_τ in the limit as $\epsilon \to 0$. We can expand $V(C_\tau, \epsilon, A)$ in powers of ϵ as follows:

$$V(C_\tau, \epsilon, A) = V_0 + \epsilon \left.\frac{\partial V}{\partial \epsilon}\right|_{\epsilon=0} + \mathcal{O}\left(\epsilon^2\right).$$

Since the contribution of C_τ to welfare in the current period is first order in ϵ, we care only about the part of $V(C_\tau, \epsilon, A)$

[10] Examination of the proof of Proposition 3 shows that it can be extended to this case.

which is also first order in ϵ, and which depends on C_τ. Computing the derivative, evaluating at $\epsilon = 0$, and keeping only the terms that depend on C_τ, we find that

$$\left.\frac{\partial V}{\partial \epsilon}\right|_{\epsilon=0} \sim -C_\tau A(AS_\tau)^{-\eta}\sum_i \frac{y_i}{\delta_i + (\eta-1)(r-A)}.$$

Thus, in the limit as $\epsilon \to 0$, C_τ must be chosen such that

$$C_\tau = \arg\max\left[\frac{(C_\tau)^{1-\eta}}{1-\eta} - C_\tau A(AS_\tau)^{-\eta}\sum_i \frac{y_i}{\delta_i + (\eta-1)(r-A)}\right]$$
$$\Rightarrow C_\tau = AS_\tau[A\sum_i y_i(\delta_i + (\eta-1)(r-A))^{-1}]^{-1/\eta}. \quad (28)$$

In equilibrium, $C_\tau = AS_\tau$, so the equilibrium condition for A is:

$$\sum_i y_i \frac{A}{\delta_i + (\eta-1)(r-A)} = 1 \quad (29)$$

and in addition, we require that A satisfies the inequalities in Eq. (27). When $\eta = 1$, we have an explicit solution:

$$A = \left[(\vec{\delta})^{-1}\cdot\vec{y}\right]^{-1}. \quad (30)$$

To verify that the equilibrium exists and is unique for $\eta > 1$, we must show that Eq. (29) always has a unique solution when $A < r + \frac{\min_i \delta_i}{\eta-1}$. When $\eta > 1$ the left hand side of Eq. (29) is a continuous increasing function of A when $0 < A < r + \frac{\min_i \delta_i}{\eta-1}$, and is equal to zero at $A = 0$. Thus, a necessary and sufficient condition for a unique equilibrium to exist is for the left hand side of Eq. (29) to be greater than 1 when $A = r + \frac{\min_i \delta_i}{\eta-1}$. Substituting, we find that this requires

$$\left(r + \frac{\min_i \delta_i}{\eta-1}\right)\sum_i \frac{y_i}{\delta_i - \min_i \delta_i} > 1. \quad (31)$$

Since $y_i > 0$ for all i, this condition is always satisfied, as the left hand side of the inequality is always equal to $+\infty$. Thus, a unique equilibrium exists.

For $\eta < 1$, the left hand side of Eq. (29) is no longer generically a monotone function of A when $A > r - \frac{\min_i \delta_i}{1-\eta}$, and may be always above 1. Thus, the equilibrium may not exist.

2. Straightforward calculations show that the equilibrium consumption path implemented by utilitarian committees will be

$$C_t = S_0 A \exp\left[-(A-r)(t-\tau)\right]. \quad (32)$$

The optimal consumption path of a single agent with discount rate δ can be found using standard methods from optimal control. The Hamiltonian for such an agent's intertemporal optimization problem is:

$$H = U(C_t)e^{-\delta(t-\tau)} + \lambda_t(rS_t - C_t) \quad (33)$$

where λ_t is the shadow price of the resource, and S_t evolves according to

$$\dot{S}_t = rS_t - C_t.$$

A standard application of the Maximum principle yields

$$(C_t)^{-\eta}e^{-\delta_i(t-\tau)} = \lambda_t$$
$$\dot{\lambda}_t = -r\lambda_t.$$

Solving the equation for λ_t, we have

$$C_t = \left[\frac{1}{\lambda_\tau}e^{-(\delta-r)(t-\tau)}\right]^{1/\eta} \quad (34)$$

where λ_τ is the initial shadow price at $t = \tau$, which we need to solve for. With this solution, we can write the evolution equation for the stock at an optimum as:

$$\dot{S_t} - rS_t = -\left(\frac{1}{\lambda_\tau}e^{-(\delta-r)(t-\tau)}\right)^{1/\eta}.$$

Multiplying through by an integration factor e^{-rt} and integrating from τ to t:

$$S_t e^{-rt} - S_\tau e^{-r\tau} = -\int_\tau^t e^{-rt}\left(\frac{1}{\lambda_\tau}e^{-(\delta-r)(t'-\tau)}\right)^{1/\eta}dt'$$

where $S_\tau = S(\tau)$ is the initial resource stock. The transversality conditions on these solutions require:

$$\lim_{t\to\infty} S_t\lambda_t = \lim_{t\to\infty} S_t\lambda_\tau e^{-r(t-\tau)} = 0.$$

Hence, the initial value of the shadow price λ_τ must satisfy:

$$S_\tau e^{-r\tau} = \int_\tau^\infty e^{-rt}\left(\frac{1}{\lambda_\tau}e^{-(\delta-r)(t-\tau)}\right)^{1/\eta}dt$$

from which we find

$$\lambda_\tau = \begin{cases} \left(\frac{\eta}{S_\tau}\frac{1}{\delta+(\eta-1)r}\right)^\eta & \text{if } \delta + (\eta-1)r > 0 \\ \infty & \text{if } \delta + (\eta-1)r < 0. \end{cases} \quad (35)$$

If $\delta + (\eta-1)r < 0$, there is no optimal path. When $\eta \geq 1$, as assumed throughout the paper, a unique optimal path exists. Thus in this case, the optimal consumption path of a single agent with discount rate δ is given by

$$C_t^\delta = S_0\left[\frac{\delta + (\eta-1)r}{\eta}\right]\exp\left(-\left(\left[\frac{\delta+(\eta-1)r}{\eta}\right] - r\right)(t-\tau)\right). \quad (36)$$

Comparing Eq. (36) to Eq. (32), we see that these two consumption paths coincide if we define the effective discount rate in the game equilibrium consumption path to be $\hat{\delta} \equiv r + \eta(A-r)$.

3. Committee member i's opinion about social welfare in equilibrium in period τ is given by

$$V_{i\tau} = \int_\tau^\infty U(C_t)e^{-\delta_i(t-\tau)}dt.$$

As Eq. (32) shows, the consumption path implemented by the utilitarian committee is

$$C_t = S_\tau A \exp\left[-(A-r)(t-\tau)\right].$$

Substituting into the previous expression and performing the integral yields the result.

Appendix B. Proposition 2

From Eq. (11), we have

$$\frac{\partial\hat{\delta}}{\partial r} = 1 + \eta\left(\frac{\partial A}{\partial r} - 1\right). \tag{37}$$

To compute $\frac{\partial A}{\partial r}$, implicitly differentiate Eq. (8) with respect to r to find

$$\sum_i y_i \frac{\frac{\partial A}{\partial r}}{\delta_i + (\eta-1)(r-A)} + (\eta-1)\left(\frac{\partial A}{\partial r} - 1\right)\sum_i y_i \frac{A}{[\delta_i + (\eta-1)(r-A)]^2} = 0.$$

Multiplying this equation through by A, using the equilibrium condition Eq. (8) for A, and defining

$$K = \sum_i y_i \frac{A^2}{[\delta_i + (\eta-1)(r-A)]^2},$$

we have

$$\frac{\partial A}{\partial r} \times 1 + (\eta-1)\left(\frac{\partial A}{\partial r} - 1\right)K = 0$$
$$\Rightarrow \frac{\partial A}{\partial r} = \frac{(\eta-1)K}{1+(\eta-1)K}.$$

Substituting this result into Eq. (37), we find

$$\frac{\partial\hat{\delta}}{\partial r} = \frac{\eta-1}{1+(\eta-1)K}(K-1).$$

Now by Jensen's inequality,

$$K = \sum_i y_i \frac{A^2}{[\delta_i + (\eta-1)(r-A)]^2} > \left(\sum_i y_i \frac{A}{\delta_i + (\eta-1)(r-A)}\right)^2 = 1$$

where the last line follows from Eq. (8), and the inequality is strict since we assume there exists $\delta_i \neq \delta_j$, with $y_i, y_j > 0$. Thus, we conclude that $\frac{\partial\hat{\delta}}{\partial r} > 0$ when $\eta > 1$.

Appendix C. Proof of Proposition 3

We begin with a useful lemma:

Lemma 1. *Consider the function*

$$W(C_\tau) = a\frac{C_\tau^{1-\eta}}{1-\eta}\Delta\tau + \frac{b}{1-\eta}((1+r\Delta\tau)S_\tau - C_\tau\Delta\tau)^{1-\eta} \tag{38}$$

where $a, b, \eta, C_\tau, S_\tau > 0$. In the limit as $\Delta\tau \to 0$, $W(C_\tau)$ is single-peaked in C_τ.

Proof.

$$\frac{\partial W}{\partial C_\tau} = aC_\tau^{-\eta}\Delta\tau - b\Delta\tau((1+r\Delta\tau)S_\tau - C_\tau\Delta\tau)^{-\eta},$$
$$\frac{\partial^2 W}{\partial C_\tau^2} = -\eta a C_\tau^{-\eta-1}\Delta\tau - \eta b(\Delta\tau)^2((1+r\Delta\tau)S_\tau - C_\tau\Delta\tau)^{-\eta-1} < 0.$$

In the limit as $\Delta\tau \to 0$, $W(C_\tau)$ has an extremum at

$$C_\tau^* = \left(\frac{a}{b}\right)^{1/\eta} S_\tau > 0.$$

Since $W(C_\tau)$ is strictly concave, C_τ^* is a global maximum, and $W(C_\tau)$ is single-peaked. □

The proof of the main result is by induction. As in our treatment of the game equilibrium for utilitarian committees, we consider a finite horizon model in which the game stops at time T, and take the limit as $T \to \infty$. In time period $\tau = T - \Delta\tau$, just before T, a member with discount rate δ_i has preferences over consumption C_t given by

$$P_{T-\Delta\tau}(C_{T-\Delta\tau}, \delta_i) = U(C_{T-\Delta\tau})\Delta\tau + e^{-\delta_i\Delta\tau}U(S_T) \tag{39}$$

where $S_T = (1 + r\Delta\tau)S_{T-\Delta\tau} - C_{T-\Delta\tau}\Delta\tau$. Substituting the expression for S_T into $P_{T-\Delta\tau}(C_{T-\Delta\tau}, \delta_i)$, it is immediate that $P_{T-\Delta\tau}(C_{T-\Delta\tau}, \delta_i)$ is of the form in Eq. (38), and hence is single-peaked in the $\Delta\tau \to 0$ limit. A vote at $T - \Delta\tau$ thus results in the median member's optimal value of current consumption being chosen. Now suppose that the median member's preferred consumption plan is chosen for all $t \in [\tau + \Delta\tau, T]$, where $\Delta\tau$ is very small. We prove that it will be chosen at τ too.

The derivation of the optimal consumption path for an agent with discount rate δ in part 2 of Appendix A can be modified to a finite horizon problem, that terminates at $\tau = T$. Imposing the boundary condition $S(T) = 0$, it is straightforward to show that if the consumption path followed from $\tau \in [\tau + \Delta\tau, T]$ is the optimal path of the median agent, we have:

$$C_t^m = S_{\tau+\Delta\tau}\left[\frac{\delta_m + (\eta-1)r}{\eta}\right]\frac{e^{-\left(\frac{\delta_m - r}{\eta}\right)(t-(\tau+\Delta\tau))}}{1 - e^{-\left(\frac{\delta_m+(\eta-1)r}{\eta}\right)(T-(\tau+\Delta\tau))}},$$

where again

$$S_{\tau+\Delta\tau} = (1+r\Delta\tau)S_\tau - C_\tau\Delta\tau. \tag{40}$$

A member with discount rate δ_i thus has preferences over current consumption C_τ given by

$$P_\tau(C_\tau, \delta_i) = \frac{C_\tau^{1-\eta}}{1-\eta}\Delta\tau + \int_{\tau+\Delta\tau}^{\infty}\frac{1}{1-\eta}(C_t^m)^{1-\eta}e^{-\delta_i(t-(\tau+\Delta\tau))}dt$$
$$= \frac{C_\tau^{1-\eta}}{1-\eta}\Delta\tau + \frac{K'}{1-\eta}[(1+r\Delta\tau)S_\tau - C_\tau\Delta\tau]^{1-\eta},$$

where

$$K' = \int_{\tau+\Delta\tau}^{T}\left[\left[\frac{\delta_m + (\eta-1)r}{\eta}\right]\frac{e^{-\left(\frac{\delta_m-r}{\eta}\right)(t-(\tau+\Delta\tau))}}{1-e^{-\left(\frac{\delta_m+(\eta-1)r}{\eta}\right)(T-(\tau+\Delta\tau))}}\right]^{1-\eta} e^{-\delta_i(t-(\tau+\Delta\tau))}dt > 0.$$

By Lemma 1, we thus again know that $P_\tau(C_\tau, \delta_i)$ is single-peaked in the $\Delta\tau \to 0$ limit. The result follows by induction, and by taking the limit as $T \to \infty$.

Appendix D. Proof of Proposition 4

Since the proof of this result is no more difficult with an arbitrary utility function $U(C_t)$, we will do it in general. Let the optimal public consumption plan of an agent with discount rate δ_i be $C(\delta_i) = (C_t^{\delta_i})_{t\geq\tau_0}$. Since individual members' preferences are time consistent, we can prove the result for e.g. $\tau = 0$, and be sure that it will then hold for all τ. Thus, $C(\delta_i)$ is the solution of

$$\max_{C_t} \int_0^\infty U(C_t)\, e^{-\delta_i t} dt \text{ s.t. } \dot{S}_t = rS_t - C_t. \tag{41}$$

We are interested in members' preferences over the set of optimal plans $\{C(\delta_i)\}$. We begin with a lemma:

Lemma 2. *Suppose that:*

1. *Initial optimal consumption C_0^δ is an increasing function of δ.*
2. *Each pair of consumption paths $\{C(\delta), C(\delta')\}$ has exactly one intersection point, i.e. for any $\delta' > \delta$, there exists a time T such that*

$$\forall t > 0, (T-t)\left(C_t^{\delta'} - C_t^{\delta}\right) > 0.$$

Then all members have single-peaked preferences over optimal consumption paths.

Proof. Let the optimal consumption path for an individual with discount rate δ be $C(\delta) = (c_t^\delta)_{t\geq 0}$. Denote the preferences over consumption paths of a member with discount rate δ by $\prec_\delta$.

We first prove that under the conditions of the lemma, given any pair of discount rates $\delta' < \delta''$, for any $\delta < \delta'$ we must have $C(\delta'') \prec_\delta C(\delta')$, and for any $\delta > \delta''$, we must have $C(\delta') \prec_\delta C(\delta'')$. Consider the case $\delta > \delta''$, and let $\delta = \delta'' + \epsilon$, where $\epsilon > 0$. We will evaluate the difference in welfare for a member with discount rate δ under the two consumption paths $C(\delta')$ and $C(\delta'')$. Let T be the intersection point of the two consumption streams. We have:

$$\begin{aligned}
&\int_0^\infty U\left(C_t^{\delta''}\right) e^{-\delta t} dt - \int_0^\infty U\left(C_t^{\delta'}\right) e^{-\delta t} dt \\
&= \int_0^T \left[U(C_t^{\delta''}) - U(C_t^{\delta'})\right] e^{-\delta t} dt - \int_T^\infty \left[U\left(C_t^{\delta'}\right) - U\left(C_t^{\delta''}\right)\right] e^{-\delta t} dt \\
&= \int_0^T \left[U\left(C_t^{\delta''}\right) - U\left(C_t^{\delta'}\right)\right] e^{-\delta'' t} e^{-\epsilon t} dt - \int_T^\infty \left[U\left(C_t^{\delta'}\right) - U\left(C_t^{\delta''}\right)\right] e^{-\delta'' t} e^{-\epsilon t} dt \\
&\geq e^{-\epsilon T} \int_0^T \left[U\left(C_t^{\delta''}\right) - U\left(C_t^{\delta'}\right)\right] e^{-\delta'' t} dt - e^{-\epsilon T} \int_T^\infty \left[U\left(C_t^{\delta'}\right) - U\left(C_t^{\delta''}\right)\right] e^{-\delta'' t} dt \\
&= e^{-\epsilon T} \int_0^\infty \left[U\left(C_t^{\delta''}\right) - U\left(C_t^{\delta'}\right)\right] e^{-\delta'' t} dt \\
&\geq 0.
\end{aligned}$$

The last inequality follows from the optimality of $C(\delta'')$. Thus, we have shown that for any $\delta > \delta''$, $C(\delta') \prec_\delta C(\delta'')$. A similar argument shows that for any $\delta < \delta'$, we must have $C(\delta'') \prec_\delta C(\delta'')$.

It is straightforward to see that with these properties in hand, members' preferences over consumption paths that are optimal for some discounted utilitarian agent must be single-peaked. Consider two paths $C(\delta')$ and $C(\delta'')$, and a member with discount rate δ, where $\delta < \delta' < \delta''$. From the above properties, we must have $C(\delta'') \prec_\delta C(\delta')$. Similarly, for any paths $C(\delta')$ and $C(\delta'')$ with $\delta'' < \delta' < \delta$, we must have $C(\delta'') \prec_\delta C(\delta')$. Thus, all members' preferences are single-peaked. □

We now show that the conditions of this lemma will always be satisfied. Theorem 2 in Becker (1983) shows that optimal initial consumption is an increasing function of δ for any concave production function. As $t \to \infty$, the path $C(\delta)$ tends to zero if $\delta > r$, and $+\infty$ if $\delta < r$. Thus, the limiting value of C_t^δ is non-increasing in δ. In addition, all optimal consumption paths are monotonic functions of time (see e.g. Kamien and Schwartz, 1991). All pairs of optimal consumption paths must therefore cross exactly once. The two conditions of Lemma 2 are thus satisfied, and members have single-peaked preferences over optimal plans.[11]

Since the voting equilibrium is the optimal plan for a member with discount rate δ_m, and the equilibrium implemented by a utilitarian committee coincides with the optimal plan of a discounted utilitarian agent with discount rate $\hat{\delta}$, members have single-peaked preferences over these plans. Since members have single-peaked preferences over optimal plans, the classic results of Black (1948) show that the plan corresponding to δ_m is the unique Condorcet winner in the set of optimal plans. Proposition 1 also showed that any equilibrium implemented by a utilitarian committee is observationally equivalent to the optimal plan of a discounted utilitarian agent with discount rate $\hat{\delta}$. Thus, a majority of agents will always prefer the voting equilibrium to the equilibrium implemented by any utilitarian committee.

Appendix E. Proof of Proposition 5

1. Let C_t^δ be the optimal plan of a single agent with discount rate δ, and define

$$V_{i\tau}(\delta) = \int_\tau^\infty U\left(C_t^\delta\right) e^{-\delta_i(t-\tau)} dt.$$

To prove the result it is sufficient to show that $V_{i\tau}(\delta)$ is a strictly concave function of δ with a unique maximum for all i. If we know this $W_\tau = \sum_i y_i V_{i\tau}$ must also be strictly concave with a unique maximum, and is thus single-peaked. The uniqueness of the maximum of $V_{i\tau}(\delta)$ follows trivially from the fact that there exists a δ such that C_t^δ is the unconstrained first best for member i, so we focus on proving concavity. From Eq. (36), we have

$$C_t^\delta = S_\tau B(\delta) e^{-(B(\delta)-r)(t-\tau)},$$

where $B(\delta) = \frac{1}{\eta}(\delta + (\eta-1)r)$. Since $B(\delta)$ is linearly increasing in δ, it suffices to show that $V_{i\tau}$ is strictly concave in B. We have

$$\begin{aligned}
V_{i\tau}(B) &= \frac{(S_\tau B)^{1-\eta}}{1-\eta} \int_\tau^\infty e^{-(1-\eta)(B-r)(t-\tau)} e^{-\delta_i(t-\tau)} dt \\
&\propto \frac{B^{1-\eta}}{1-\eta} \frac{1}{\delta_i + (\eta-1)(r-B)},
\end{aligned}$$

where we have assumed that $B < \min_i(r + \frac{\delta_i}{\eta-1})$, so that the integral converges for all i.[12]

[11] The proof is no more difficult for an arbitrary concave production function $F(S)$ that admits an interior steady state. In this case the steady state value of consumption on a path $C(\delta)$ is given by $F((F')^{-1}(\delta))$, which by the concavity of F, is again a non-increasing function of δ. The rest of the proof goes through unchanged.

[12] This is without loss of generality since if any agents' welfare becomes $-\infty$, W_τ will also be $-\infty$. We can thus exclude these paths from our analysis.

Differentiating twice with respect to B, ignoring common factors, and defining $v = \delta_i + (\eta - 1)(r - B)$, one can show that

$$\frac{\partial^2 V_{i\tau}}{\partial B^2} \propto 2(\eta-1)vB - \eta v^2 - 2(\eta-1)B^2$$
$$= -\eta(\eta^2-1)B^2 + 2(\eta^2-1)(\delta_i + (\eta-1)r)B - \eta(\delta_i + (\eta-1)r)^2.$$

The last line is a quadratic in B, and is strictly negative when $B = 0$. The discriminant of the quadratic is

$$4(\delta_i + (\eta-1)r)^2(1-\eta^2),$$

and is less than or equal to zero when $\eta \geq 1$. Thus, when $\eta \geq 1$ the quadratic has no real roots ($\eta > 1$), or a single root ($\eta = 1$). Thus, we conclude that for $\eta > 1$, $\frac{\partial^2 V_{i\tau}}{\partial B^2} < 0$ for all relevant values of B. When $\eta = 1$, $\frac{\partial^2 V_{i\tau}}{\partial B^2} < 0$ everywhere except at a single value of B, and thus $V_{i\tau}$ is still single-peaked in B.

2. By part 1 of this proposition, we know that δ^* exists and is unique and may be determined by solving for the value of B that maximizes

$$W(B) = \sum_i y_i \frac{(S_\tau B)^{1-\eta}}{1-\eta} \frac{1}{\delta_i + (\eta-1)(r-B)}. \quad (42)$$

If B^* is the solution of this problem, from our discussion of Eq. (36) we know that

$$\delta^* = r + \eta(B^* - r). \quad (43)$$

Since B^* does not depend on S_τ, we know that δ^* is independent of τ. Similarly, from Proposition 1 we know that

$$\hat{\delta} = r + \eta(A - r)$$

where A satisfies the equilibrium condition (8). Thus, to show that $\delta^* < \hat{\delta}$, we must show that $B^* < A$.
Since by part 1 of this proposition $W(B)$ is concave in B, showing that $B^* < A$ is equivalent to showing that

$$\left.\frac{\partial W(B)}{\partial B}\right|_{B=A} < 0.$$

Differentiate Eq. (42) with respect to B to find that:

$$\frac{\partial W}{\partial B} = \frac{S_\tau^{1-\eta}}{B^{1+\eta}} \sum_i y_i \left[\frac{B}{\delta_i + (\eta-1)(r-B)} - \frac{B^2}{(\delta_i + (\eta-1)(r-B))^2}\right].$$

Evaluating at $B = A$, and using the equilibrium condition (8), we have

$$\left.\frac{\partial W(B)}{\partial B}\right|_{B=A} \propto \sum_i y_i \left[1 - \frac{A^2}{(\delta_i + (\eta-1)(r-A))^2}\right].$$

Since

$$\sum_i y_i \frac{A^2}{(\delta_i + (\eta-1)(r-A))^2} > \left(\sum_i y_i \frac{A}{(\delta_i + (\eta-1)(r-A))}\right)^2 = 1$$

we conclude that $\left.\frac{\partial W(B)}{\partial B}\right|_{B=A} < 0$. The result follows.

Appendix F. Proof of Proposition 6

From Eq. (36) in Appendix A, we know that with $\eta = 1$ the optimal consumption path of an agent with discount rate δ is:

$$C_t^\delta = S_\tau \delta \exp(-(\delta - r)(t - \tau)).$$

The value of this path to the utilitarian committee at τ is:

$$W_\tau(\delta) = \sum_i y_i \int_\tau^\infty \ln(C_t^\delta) e^{-\delta_i(t-\tau)} dt.$$

Substituting for C_t^δ and solving for the value of δ that maximizes $W_\tau(\delta)$ shows that

$$\delta^* = \frac{\left[\vec{y} \cdot \vec{\delta}^{-1}\right]}{\left[\vec{y} \cdot \vec{\delta}^{-2}\right]}.$$

From the results in Proposition 1, voting dominates the utilitarian committee's equilibrium consumption plan in all periods iff

$$\sum_j y_j \left[\delta_j^{-1} \ln \delta_m - \delta_j^{-2} \delta_m\right] - \sum_j y_j \left[\delta_j^{-1} \ln \left[\vec{y} \cdot \vec{\delta}^{-1}\right]^{-1} - \delta_j^{-2} \left[\vec{y} \cdot \vec{\delta}^{-1}\right]^{-1}\right] > 0$$
$$\iff \frac{\left[\vec{y} \cdot \vec{\delta}^{-1}\right]}{\left[\vec{y} \cdot \vec{\delta}^{-2}\right]} \ln \frac{\left[\vec{y} \cdot \vec{\delta}^{-1}\right]^{-1}}{\delta_m} < \left[\left[\vec{y} \cdot \vec{\delta}^{-1}\right]^{-1} - \delta_m\right].$$

Substituting the definitions of δ^* and $\hat{\delta}$ yields the result.

References

Arrow, K., Cropper, M., Gollier, C., Groom, B., Heal, G., Newell, R., Nordhaus, W., Pindyck, R., Pizer, W., Portney, P., Sterner, T., Tol, R.S.J., Weitzman, M., 2013. Determining benefits and costs for future generations. Science 341 (6144), 349–350.
Arrow, K.J., 1999. Discounting, morality, and gaming. In: Portney, P.R., Weyant, J.P. (Eds.), Discounting and Intergenerational Equity. Resources for the Future.
Barbera, S., Jackson, M.O., 2004. Choosing how to choose: self-stable majority rules and constitutions. Q. J. Econ. 119 (3), 1011–1048.
Becker, R.A., 1983. Comparative dynamics in the one-sector optimal growth model. J. Econ. Dyn. Control. 6, 99–107.
Black, D., 1948. On the rationale of group decision-making. J. Polit. Econ. 56 (1), 23–34.
Dasgupta, P., 2001. Human Well-being and the Natural Environment. Oxford University Press, Oxford.
Dasgupta, P., Maskin, E., 2008. On the robustness of majority rule. J. Eur. Econ. Assoc. 6 (5), 949–973.
Downs, A., 1957. An Economic Theory of Democracy. Harper Collins, New York.
Drupp, M., Freeman, M.C., Groom, B., Nesje, F., 2018. Discounting disentangled. Am. Econ. J. Econ. Pol. forthcoming.
Feldstein, M.S., 1964. The social time preference discount rate in cost benefit analysis. Econ. J. 74 (294), 360.
Freeman, M.C., Groom, B., 2015. Positively gamma discounting: combining the opinions of experts on the social discount rate. Econ. J. 125 (585), 1015–1024.
Gollier, C., 2012. Pricing the Planet's Future: The Economics of Discounting in an Uncertain World. Princeton University Press.
Gollier, C., 2016. Gamma discounters are short-termist. J. Public Econ. 142, 83–90.
Gollier, C., Hammitt, J.K., 2014. The long-run discount rate controversy. Ann. Rev. Resour. Econ. 6 (1), 273–295.
Gollier, C., Zeckhauser, R., 2005. Aggregation of heterogeneous time preferences. J. Polit. Econ. 113 (4), 878–896.
Hammond, P., 1996. Consequentialist decision theory and utilitarian ethics. In: Farina, F., Hahn, F., Vannucci, S. (Eds.), Ethics, Rationality, and Economic Behaviour. Clarendon Press, Oxford, pp. 92–118.
Heal, G.M., Millner, A., 2014. Agreeing to disagree on climate policy. Proc. Natl. Acad. Sci. 111 (10), 3695–3698.
Jackson, M.O., Yariv, L., 2015. Collective dynamic choice: the necessity of time inconsistency. Am. Econ. J. Microecon. 7 (4), 150–178.
Jouini, E., Marin, J.-M., Napp, C., 2010. Discounting and divergence of opinion. J. Econ. Theory 145 (2), 830–859.
Kamien, M.I., Schwartz, N.L., 1991. Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management. 2nd ed., Elsevier Science.
Krusell, P., Kuruşçu, B., Smith, A.A., Jr, 2002. Equilibrium welfare and government policy with quasi-geometric discounting. J. Econ. Theory 105 (1), 42–72.

Krusell, P., Smith, A., 2003. Consumption-savings decisions with quasi-geometric discounting. Econometrica 71 (1), 365–375.

Laibson, D., 1994. Self-control and Saving. Technical report, Harvard University. (unpublished manuscript).

Laibson, D., 1997. Golden eggs and hyperbolic discounting. Q. J. Econ. 112 (2), 443–478.

Marglin, S.A., 1963. The social rate of discount and the optimal rate of investment. Q. J. Econ. 77 (1), 95–111.

May, K.O., 1952. A set of independent necessary and sufficient conditions for simple majority decision. Econometrica 20 (4), 680–684.

Meltzer, A.H., Richard, S.F., 1981. A rational theory of the size of government. J. Polit. Econ. 89 (5), 914–927.

Millner, A., 2018. Non-dogmatic Social Discounting. Working Paper.

Millner, A., Heal, G., 2018. Time consistency and time invariance in collective intertemporal choice. J. Econ. Theory 176, 158–169.

Nordhaus, W.D., 2008. A question of balance. Yale University Press.

Phelps, E.S., Pollak, R.A., 1968. On second-best national saving and game-equilibrium growth. Rev. Econ. Stud. 35 (2), 185–199.

Ramsey, F.P., 1928. A mathematical theory of saving. Econ. J. 38 (152), 543–559.

Sen, A., 2017. Collective Choice and Social Welfare: Expanded Edition. Penguin.

Stern, N.H., 2007. The Economics of Climate Change : The Stern Review. Cambridge University Press, Cambridge.

Strotz, R.H., 1955. Myopia and inconsistency in dynamic utility maximization. Rev. Econ. Stud. 23 (3), 165.

Weitzman, M.L., 1976. On the welfare significance of national product in a dynamic economy. Q. J. Econ. 90 (1), 156–162.

Weitzman, M.L., 1998. Why the far-distant future should be discounted at its lowest possible rate. J. Environ. Econ. Manag. 36 (3), 201–208.

Weitzman, M.L., 2001. Gamma discounting. Am. Econ. Rev. 91 (1), 260–271.